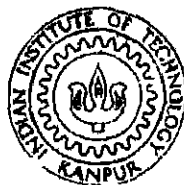


# SEEPAGE THROUGH PARTIALLY LINED CHANNELS

*by*

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MARCH, 1986

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# SEEPAGE THROUGH PARTIALLY LINED CHANNELS

*A Thesis Submitted*  
in Partial Fulfilment of the Requirements  
for the Degree of

MASTER OF TECHNOLOGY

*by*  
KATHAWALA SARFRAZ HUSSAIN

*to the*  
DEPARTMENT OF CIVIL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
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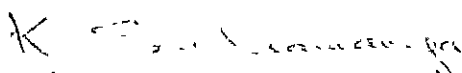
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C E R T I F I C A T E

The thesis ' SEEPAGE THROUGH PARTIALLY LINED CHANNELS' by Kathawala Sarfraz Hussain is hereby approved as a creditable report on research carried out and presented in a manner which warrants its acceptance as a prerequisite for the degree of MASTER OF TECHNOLOGY. The work has been carried out under my supervision and has not been submitted elsewhere for the degree.

March, 1986

  
( DR. K. SUBRAMANYA )  
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S.H. KATHAWALA

## ABSTRACT

Numerical analysis of two dimensional steady state unconfined seepage through partially lined channels has been presented. The channels are either lined at base only and sides unlined or sides only lined and base unlined. The soil is considered to be homogenous and isotropic while the subsoil was either impermeable ( condition B ) or infinitely permeable ( condition A ). The parameters governing seepage are the depth of water in the channel, the base width of the channel, the position of water table and the position of subsoil layer. The side slope of the channel is kept constant at 1H:1V. The condition A is a more active state of seepage than condition B. There is a marginal effect of position of water table on the percentage reduction in discharge for both cases of lining for condition B while for condition A the percentage reduction in seepage is found to increase with the lowering of the water table for base only lined case and decrease for sides only lined case. The lowering of the impermeable layer increased the percentage reduction in seepage for both cases of lining for condition B while lowering of the permeable layer decreased the percentage reduction in seepage for condition A. For  $H_w/W_b$  less than 0.4 for a set of other parameters the base lining only of the channel was found to be more effective in reducing seepage

while for  $H_w/W_b$  greater than 0.4 sides lining was found to be more effective for condition A. It is also observed that the percentage reduction in seepage due to lining is not linearly related to the percentage of the total wetted perimeter lined. The analysis is done with the help of a computer program based on finite element method.

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## LIST OF SYMBOLS

$a$	Constant
$A$	Area of element
$b$	Constant
$c$	Constant
$d$	Average pore diameter
$D$	Any domain
$D_i$	Position of impermeable layer
$D_p$	Position of permeable layer
$D_w$	Position of water table
$e$	Suffix denoting element
$f_L$	Lining factor
$h$	Elevation above datum
$H_w$	Depth of water in the channel
$i$	Suffix, gradient
$I$	Functional
$I_s$	Seepage per unit top width of the channel
$k$	Suffix, coefficient of permeability
$L_1, L_j, L_k$	Area co-ordinates
$m$	Side slope of trapezoidal channel
$n$	Total number of elements
$n_x, n_y$	Direction cosines
$N$	Shape function
$p$	Pressure
$P$	Any point within a triangular element
$q$	Capacity of source or sink/area
$Q$	Seepage rate
$\bar{Q}$	Capacity of source or sink
$R_e$	Reynolds number
$R^{(e)}$	Residual
$s$	Elemental distance
$v$	Seepage velocity

$\bar{V}$	Velocity
$v_x, v_y, v_z$	Velocities in x and y directions
$W_b$	Width of base
$W_s$	Width of water surface
$x$	Coordinate direction
$x_1$	Principal axis of tensor
$y$	Coordinate direction
$y_1$	Principal axis of tensor
$z$	Coordinate direction, elevation head
$z_1$	Principal axis of tensor
$\alpha$	Relaxation factor
$\beta$	Cartesian coordinate of any specific point for a super element
$\gamma$	Specific weight of water
$\delta$	$\gamma_i - \phi_i$
$\phi$	Piezometric head
$\phi_1, \phi_j, \phi_k$	Nodal heads
$\eta$	Cartesian coordinate of any specific point for a super element
$\rho$	Fluid density
$\mu$	Fluid viscosity

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## CHAPTER 1

### INTRODUCTION

#### 1.1 General:

Canals are the main links between the storage and the user of the stored water. Since there is a large area under cultivation in our country importance of irrigation cannot be overemphasized. And indeed channel design and maintenance have gained considerable attention of investigators in this field. Though canals are essential for irrigation they also cause certain environmental disturbance by the waterlogging due to excessive seepage through their perimeter.

A considerable quantity of water flowing in irrigation channels is lost to the adjacent areas through seepage. It has been estimated that transit losses in the alluvial channels in the northern India are 17 percent for the main canals and branches, 8 percent for distributaries, and 20 percent for the watercourses, which gives a total loss of 45 percent of the water entering the canal head. The necessity of lining of irrigation channels cannot, therefore, be overemphasized. Besides reducing seepage losses in channels and thereby leading to more efficient utilization, the lining prevents waterlogging and weed growth, and reduces requirement of land and maintenance cost. It has been estimated that lining of all the existing unlined channels in India could

save enough water to irrigate an additional six million hectares against the present irrigated area of 36.4 million hectares [38].

However, nowadays as a matter of policy all major channels are fully lined to eliminate the problem of water-logging due to seepage and to improve the efficiency of the channels. Therefore the main cause of concern are the remaining unlined alluvial channels built in the beginning of this century which cause heavy seepage and accompanying water logging. To mitigate the severity of the problem it is necessary to partially line their perimeter, at least as a first phase. Partial lining may also be resorted to due to paucity of funds. The present study is aimed to help in precisely such situations when one has to judge between lining the sides only or base only for partial lining of the channels.

## 1.2 Why Numerical Method of Analysis of Seepage:

Considering the importance of seepage through channels, considerable work has been done by researchers to analyse the seepage through channels. Seepage depends on a vast number of factors, foremost being the soil type. It is difficult to accurately represent the actual soil strata occurring in nature in the theoretical analysis of seepage. Therefore, most of the earlier analytical solution and analyses, done by analogy studies and sand models, tended to simplify the actual physical problem so as to make it amenable to either mathematical



manipulations or easier experimentation. Such simplifications therefore provided solutions which were merely estimates of the actual results.

Numerical techniques, which are of recent origin, are comparatively more versatile and capable of handling complex physical conditions. Though actual physical situation may be approximated to facilitated quicker results, they are essentially not simplified. Keeping in view this versatility of the technique and also the availability of the resources for the numerical analysis, it was chosen for the present study.

There are two powerful numerical analysis techniques available; namely, the Finite Difference Technique and the Finite Element (FE) Analysis. Both are quite versatile and are capable of providing quick and accurate solutions. However, one overriding advantage of the finite element analysis over the finite difference technique is its ability to quite accurately approximate the physical boundary of the region and the possibility of applying boundary condition to the whole system rather than to individual elements. Because of this advantage it is generally used for seepage analysis problems. Using the FE method, the solution of the basic equation governing two-dimensional steady state unconfined seepage through porous media is obtained for various sets of parameters affecting seepage when either only the base is lined

or only the sides are lined. The results are discussed in chapter 5.

### 1.3 Organisation of the Present Study:

The present work is presented in six chapters and an appendix. Chapter 2 deals with the brief review of the past work on seepage through channels. The literature review is presented under three heads, namely, analytical studies, analogy studies and numerical studies alongwith the limitations of each. In the numerical study category reference has been made to most of the finite element analysis for unconfined flow through porous media. The chapter ends with an emphasis on the need for the present study. Chapter 3 deals with the formulation of the governing equation for unconfined steady state seepage into the matrix form. The basic assumptions of the seepage through channels, Darcy's law and the Laplace equation, brief description of the finite element method, the eventual formulation of different matrices and application of boundary conditions are described. Generation of phreatic surface and calculation of actual seepage quantity are also discussed toward the end of the chapter. Chapter 4 is related to the description of the computer program 'CANSEP-86' used for the present study. Important subroutines of the program are discussed. An automatic Mesh Generation technique which is incorporated in the program is also described. This program was developed at I.I.T.Kanpur

by Achar [23], and certain necessary modifications were introduced for the use of the present study as described in chapter 4. Chapter 5 presents the results of the present study on seepage through partially lined channels. The verification of the program with the results of the previous workdone, the choice of the relaxation factor and optimum mesh size are discussed in the beginning of the chapter. The results of the present study are presented in the form of graphs and tables and are discussed. Chapter 6 lists the conclusions from the present work and recommendations for further work on this topic. Appendix deals with the instructions for giving the input for a given problem, two examples at the end further clarify the instructions.

## CHAPTER 2

### LITERATURE SURVEY

#### 2.1 Introduction:

Steady state two dimensional flows through porous media which obey Darcy's Law are essentially governed by the Laplace equation. The problem of seepage through channels have been tackled by a number of investigators. Most of the solutions available are analytical. Analytical solutions for partially lined canals using transformation techniques or certain analogy methods are available too. However, most of the numerical studies available on seepage are related to earth dams. Nevertheless, the basic requirement in the problems of seepage through earth dams or through channels laid in porous media is to obtain the phreatic surface which is not known a priori. Therefore, the formulations and approach to the problem in the former case can be adopted for the present study, albeit with the required changes in the boundary conditions. A brief review of the literature available on various aspects of seepage analysis using different methods of analysis is presented in this chapter.

#### 2.2 Literature Survey:

The previous work done on the subject can be broadly classified into three categories:-

- (i) Analytical studies.
- (ii) Analogy studies.
- (iii) Numerical studies.

The literature available within each category is described below.

#### 2.2.1 Analytical Studies:

Analytical solutions are available for evaluating seepage losses from channels under steady conditions for the following cases:

- i) Channels located in homogenous and isotropic medium extending upto infinite depth with shallow water table.
- ii) Channels located in homogenous and isotropic medium extending to finite depths with shallow water table.
- iii) Channels located in homogenous and isotropic medium extending upto infinite depth with very deep water table.

Theoretical solutions for steady state seepage from unlined canals of various cross sections and for different types of flow boundaries are presented by various investigators, notably Vedernikov, Polubarinova - Kochina [1], Harr [2], Muskat [3], and Hammad [4]. All these solutions are based on highly simplified physical situation which is quite amenable to mathematical manipulations.

Garg and Chawla [6] have obtained a closed solution of seepage from a trapezoidal channel in homogenous,

isotropic media extending to infinite depth with drainage at a finite distance from the channel.

Sharma and Chawla [7], using conformal mapping method gave a solution of seepage from channel in a homogenous medium extending to a finite depth to drains located at a finite distance from the channel considering horizontal and vertical drainage. In case of horizontal drainage, the streamlines of seepage flow from channel join the bottom of the drain at various points along its width. In case of vertical drainage, seepage might enter from both sides of the drain. The streamline of seepage on both flanks of the drain would reach the drain in a horizontal direction.

Lining of channels is now an accepted practice to reduce seepage losses and <sup>prevent</sup> waterlogging. Therefore partial or complete lining of channels deserve a detail study. Analytical solutions of some of the problems of partially lined trapezoidal channels using hodograph and Schwarz-Christoffel transformation techniques have been reported by Subramanya et al. [5]. The main cases studied are: i) Seepage from trapezoidal channels lined at sides only and bottom unlined, ii) Seepage from trapezoidal channels lined at bottom only and sides unlined. The medium is assumed to be homogenous, isotropic and of infinite depth. Also, the effect of water table is not taken into consideration and it is assumed that the capillary action is absent. The variation of the

seepage quantity with the aspect ratio and side slopes of the channels in the aforementioned two cases was studied. It was shown that for shallow channels, the bottom lining was more effective than side lining and resulted in about 50 percent reduction of seepage. For deeper channels it was seen that side lining was more effective.

#### Limitations of Analytical Studies:

The seepage quantity given by the theoretical formulae corresponds to steady state conditions which are difficult to attain in practice due to fluctuations of water depth in the channel and the water table. Thus except for cases where steady state conditions can be attained early as for instance in soils of high permeability with main drainage located close to channel, the discharge as computed from the theoretical formulae may be poor estimate of the likely losses and only serve to indicate the order of seepage losses for deciding the necessity or otherwise of lining.

The assumption of homogenous and isotropic soil in the complete seepage zone may not be always true. Also the boundary conditions created by presence of nearby pumping wells etc. render the application of analytical solutions trivial.

#### 2.2.2 Analogy Studies:

For solutions of practical problems such as  
(i) seepage from a channel underlain by a highly pervious

material or impervious material at a finite depth from the base of the channel, (ii) channel underlain by layered soil of different permeability and (iii) for the case of channel bed being practically impervious as a result of silting, the exact solutions are not available. In all such situations, an alternative to solve the basic governing differential equations, namely, the Laplace equation with the usual boundary conditions, is by using analogy methods like electrical analogy, sand models, electrolytic tank etc.

Analogic studies of unlined and partially lined canals for both steady and transient flow conditions using resistance network analog have been reported by Bouwer [8,9]. A study was made of the theoretical effect of depth and shape of channel and position of the ground water table on seepage from channels or streams. Soil conditions included in the analyses were uniform soil, soil underlain at varying depths by much more permeable soil, by much less permeable soil, or by a free draining layer, and a thin, slowly permeable (clogged) layer at the wetted perimeter of the channel. The contribution of unsaturated flow to seepage was examined and analysed.

In actuality, seepage is a dynamic process that is complicated by the non-uniformity of soil, by the effect of water quality, erosion, sedimentation, biological activity, etc., on soil hydraulic conductivity, by fluctuating water



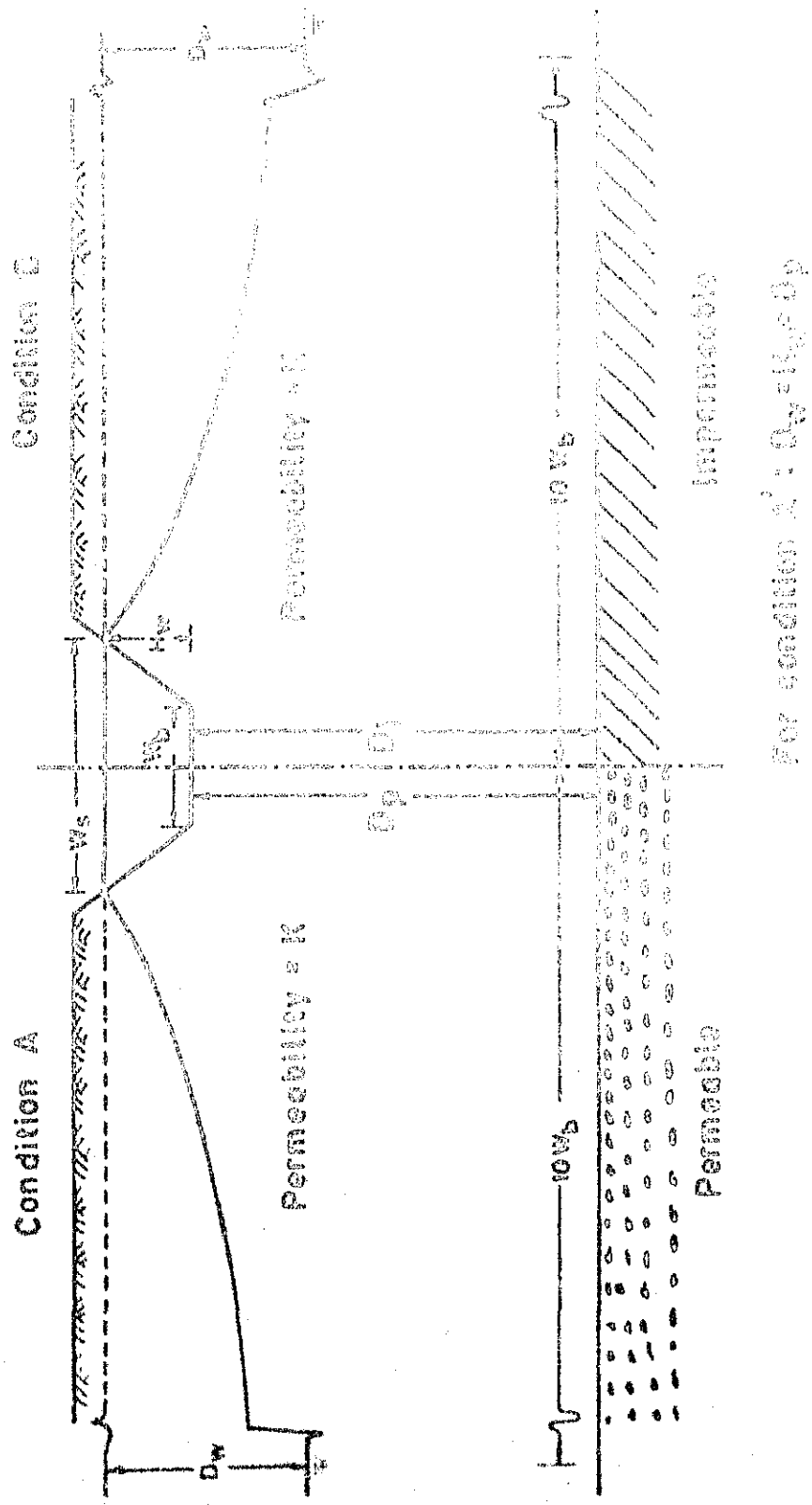


FIG. 2.1 DEFINITION SKETCH FOR SEEPAGE CONDITIONS A, B AND C

levels in the channel and in the ground, periodic dry ups of the channel , etc. For an analysis of how seepage is affected by conditions of channel, soil and groundwater table, etc. simplifications were introduced and the following basic seepage models were distinguished ( Refer Fig. 2.1):

Condition A- Seepage into uniform soil underlain by material of very high hydraulic conductivity. If the water table is at or below the top of the permeable underlying material, the condition reduces to the case of seepage to drainage layer . The condition has received considerable attention in literature and is referred to as condition A'.

Condition B - Seepage into uniform soil underlain by material of much lower hydraulic conductivity.

It may be noted that the same seepage models have been considered for the present study.

In the context of the present study it will be worthwhile to mention the important observations of Bouwer[11,12] regarding the seepage through partially lined canals, though not all of them have been substantiated with actual experiments. According to Bouwer, lining of channel banks alone did not have a significant effect on the water table position, and thus not on seepage, in a viscous flow model study of seepage under condition B with relatively large  $D_1$  [13] . For smaller values of  $D_1$ , more significant seepage reductions can be expected.

Lining of the channel bottom alone does not result in appreciable seepage reductions, except for small values of  $H_w$  and/or  $D_p$  when most of the original seepage already takes place through the bottom. Also, the presence of free draining layers at relatively small depths below channel bottom can help the effectiveness of the bottom lining of a channel.

However, there was no literature available which substantiated with experimental or theoretical data the important observations discussed above and therefore in the present study it has been attempted to establish the validity of fact.

Other analogy studies in the related field are the study of seepage from leveed rivers into low lying adjoining lands by Todd and Bear [14] with a tank electric analog and study of seepage through partially lined canals using electrolytic tank by L.R.I.P.R.I.[15], Amritsar.

#### Limitations of Analogy Studies:-

With analogs, anisotropy is not readily handled and transformation to an equivalent isotropic medium is necessary. Thus, the complexity of the actual physical situation is compromised.

Studies with sand models are particularly prone to scale errors and also equipment and experimental errors.

Such studies are generally confined to two dimensional problems only. It is particularly difficult to

apply diverse boundary conditions when analysis of seepage is done using resistance analog technique.

### 2.2.3 Numerical Studies:

Flow through porous media have been analysed using the numerical methods like the finite element method and the finite difference method quite extensively. The works available relate to the problem of analysis of free surface flows . Since seepage through channels also comes under the same category the literature available on free surface flows is listed alongwith the literature related to seepage.

Zienkiewicz and Cheung [16] have presented a general method of analysis of flow through porous media using the finite element technique. Taylor and Brown [17] have obtained solution for flow through porous media with free surface. They used the fluid pressure as the dependent variable and the flow rate was determined after the pressures were obtained. An approximate free surface was assumed initially and the pressures at various nodes obtained. The pressure at nodes on the free surface was checked and if it was not equal to zero or greater than the permissible error, the whole procedure was repeated after modifying the position of free surface until the pressure on free surface became zero. Other works for location of free surface are presented by Liggett [18] and Baiocchi, et al. [19].

Finn [20] presents a finite element analysis of seepage through dams. The region of steady seepage was divided into contiguous triangular elements. The nodal potential was used as the variable. The nodal potential on the free surface was compared with the elevation of the node above a datum. The procedure was repeated until the potential of the free surface node became equal to its elevation. Askew and Thatcher [21] have presented an algorithm for calculating the seepage through a porous dam using the finite element method.

Neuman and Witherspoon [22] performed an analysis of steady seepage with a free surface. Issacs [24] has reviewed various methods of adjusting the phreatic line in seepage analysis. It is concluded by him that the method used for the adjustment of phreatic line makes no significant difference to the results obtained in principal application and the choice of method depends, therefore, on its suitability for the particular application and the ease with which it can be implemented.

Das Gupta and Mustafa [31] have adapted the general purpose program, FEAP, developed by Taylor [26] to convert the package into a continuous simulation for seepage problem with a free surface boundary. The results obtained are verified with Bouwer's [11] analog study on seepage from channels.

Varoğlu and Finn [27] present a variable domain finite element analysis of seepage from a triangular ditch into permeable soil underlain at a finite depth by a drain.

A finite element analysis of seepage from a trapezoidal channel towards a ditch has been reported by Brebbia [28]. Two cases were considered; a) permeable wetted perimeter and b) permeable sides with impermeable bed. The soil system was assumed to consist of a layer of isotropic, homogeneous top soil underlain by an anisotropic layer. The base under this layer was considered impermeable. Like Bouwer [11,12], Brebbia too does not provide any data to substantiate the various observations regarding the reduction in seepage.

A solution of transient free surface flow in porous media is given by Shun, Cheng and Li [29]. The numerical scheme is applied to isotropic and anisotropic earth dam problems and also to drainage from a ditch.

Seepage flow across a discontinuity in hydraulic conductivity is presented by Issacs [30].

#### Limitation of Numerical Techniques:-

Even the most efficient finite element computer codes require a relatively large amount of computer memory and time. Hence, use of the technique is limited to those who have access to relatively large, high-speed computers.

The results of a numerical method must be interpreted with care and one must be aware of the assumptions employed in the formulation etc.

The most tedious aspect of the use of the finite element method is the basic process of subdividing the continuum and of generating error - free input data for computer. Although these processes may be automated to a degree, they have not been totally accomplished by computer because some engineering judgement must be employed in the discretization.

### 2.3 Need for Present Study:

Loss of water through seepage from irrigation channels constitutes quite a substantial percentage of the total utilizable water. It has been estimated that transit losses in the alluvial channels in the northern India are 17 percent for the main canals and branches, 8 percent for distributaries, and 20 percent for the water courses which gives a total loss of 45 percent of the water entering the canal head [38]. The necessity of lining irrigation channels cannot, therefore, be overemphasised. Besides reducing seepage losses in channels and thereby leading to more efficient utilization, the lining prevents waterlogging and weed growth and reduces requirement of land and maintenance cost. It has been estimated that lining of all the existing unlined channels in India could save enough water to irrigate an additional six million hectares against the present area of 36.4 million hectares [38].

However, the complete lining of irrigation channels involve huge costs. A practical idea would be to partly line the channel at least as a first phase, if comparative reduction is obtainable. As seepage from canals depends on a large number of variables, experimental or numerical analyses are generally preferred for obtaining seepage. As no data are available for seepage through partially lined channels, the present study is aimed to provide the same. Analysis has been performed using different values for the depth of water table, depth of pervious or impervious layer, head in the channel and either base or side lined conditions. The data obtained are presented in graphical as well as tabular form for easy reference to people actually designing the lining of channel.



## CHAPTER 3

## FORMULATION OF THE GOVERNING EQUATION

3.1 Basic Assumptions:

The basic assumptions that govern the flow through a porous medium are:

1. The flow occurs only in the saturated region of a porous soil and that the pores in a medium such as sand are interconnected and that the fluid can flow through them.
2. The flow is laminar such that the Reynolds number of the flow, given by

$$R_e = \frac{Vd\rho}{\mu} ; \text{ where } \begin{array}{ll} d = & \text{average pore diameter} \\ \mu = & \text{fluid viscosity} \\ \rho = & \text{fluid density} \\ V = & \text{velocity} \end{array}$$

is less than unity.

3. Both the fluid and porous medium are incompressible.
4. The viscosity of fluid is constant.

Other assumptions for the present study are:

- a) The flow is steady and two-dimensional i.e. the seepage zone extends upto infinity on either side of the section considered without any change in the material or boundary conditions.

- b) The bottom and side of the channel are not clogged.

### 3.2 Governing Equation:

The relationship governing ground water flow is known as Darcy's law. Its form can be deduced from the general Navier-Stokes equations under the assumption that the flow is laminar and that the inertial forces are negligible when compared with the viscous forces. Darcy's law establishes a linear relationship between the velocity of fluid flow and the hydraulic gradient. It may be stated as

$$v = -k \frac{d\phi}{ds} \quad (3.1)$$

where,  $\phi = \frac{p}{\gamma} + h$

$v$  = seepage velocity

$k$  = coefficient of permeability

$p$  = pressure

$\gamma$  = specific weight of water

$h$  = elevation above datum.

In case of isotropic soil, a symmetric seepage tensor is introduced

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} \partial\phi/\partial x \\ \partial\phi/\partial y \\ \partial\phi/\partial z \end{bmatrix} \quad (3.2)$$

If the principal axes of this tensor,  $xx$ ,  $yy$ ,  $zz$  are chosen

Eq. (3.2) becomes

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix} \begin{bmatrix} \partial\phi/\partial x \\ \partial\phi/\partial y \\ \partial\phi/\partial z \end{bmatrix} \quad (3.3)$$

When the seepage is considered as two-dimensional and the principal axes are horizontal (X-direction) and vertical (Y-direction), then

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} k_{xx} & 0 \\ 0 & k_{yy} \end{bmatrix} \begin{bmatrix} \partial\phi/\partial x \\ \partial\phi/\partial y \end{bmatrix} \quad (3.4)$$

Substituting  $v_x, v_y$  in the continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad (3.5)$$

Again substituting the values of  $v_x$  and  $v_y$  from Darcy's law, we obtain

$$\frac{\partial}{\partial x} \left( k_{xx} \frac{\partial\phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{yy} \frac{\partial\phi}{\partial y} \right) = 0 \quad (3.6)$$

For homogenous media

$$k_{xx} \frac{\partial^2\phi}{\partial x^2} + k_{yy} \frac{\partial^2\phi}{\partial y^2} = 0 \quad (3.7)$$

If there is a source or sink of capacity  $\bar{Q}$ , Eq (3.7) can be written as

$$k_{xx} \frac{\partial^2 \phi}{\partial x^2} + k_{yy} \frac{\partial^2 \phi}{\partial y^2} + \bar{Q} = 0 \quad (3.8)$$

Equation (3.8) is the governing differential equation for the two dimensional steady state ground water flow through a homogenous medium.

By the application of the principles of variational calculus, it may be shown that the function,  $\phi$ , which minimises the functional,  $I$ , where,

$$I = \iint \left[ k_{xx} \frac{\partial^2 \phi}{\partial x^2} + k_{yy} \frac{\partial^2 \phi}{\partial y^2} \right] dx dy$$

over the whole region of solution is the solution of the differential equation (3.7) .

### 3.3 Finite Element Method:

For an exact solution the value of  $\phi$  would have to be determined at each and every point within the region. In the finite element method the aim is to determine  $\phi$  at a finite number of specified points or nodes. Between the node points,  $\phi$  is assumed to vary in such a way that it can be uniquely defined in terms of nodal values. Lines may be drawn between the node points which subdivide the whole region into a number of small regions or elements. Then the assumed variation of  $\phi$  over any one element can be specified in terms of the values at nodes within or along the boundary

of the element. In the program used here, the element chosen is triangular because any shape can be simply approximated by triangles and graded meshes in which the distances between nodes vary depending on the accuracy and detail required are easily produced. Since there are only three nodes associated with each element,  $\phi$  is assumed to vary linearly within the element. The algebraic functions used to interpolate the field variable over an element are called interpolating functions.

Often polynomials are selected as interpolating functions for the field variable in view of the ease with regard to their mathematical manipulations like differentiation and integration. The degree of the polynomial chosen depends on the number of nodes assigned to the element and the nature and number of unknowns at each node. The interpolating function should be so chosen that the following general requirements are met:

a) At the element interfaces or boundaries the field variable  $\phi$  and any of its partial derivatives upto one order less than the highest order derivative appearing in the functional must be continuous. This is called 'compatibility' requirement.

b) All uniform states of  $\phi$  and its partial derivatives upto the highest order appearing in the functional  $I(\phi)$  should have representation in  $\phi^{(e)}$ , when, in the limit,

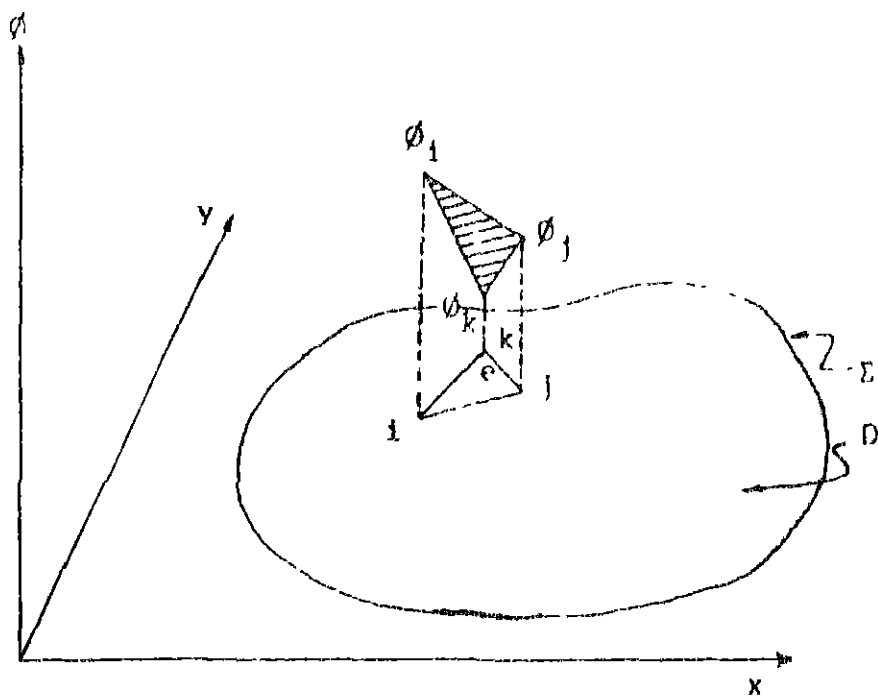


FIG. 3.1 a A DOMAIN  $D$

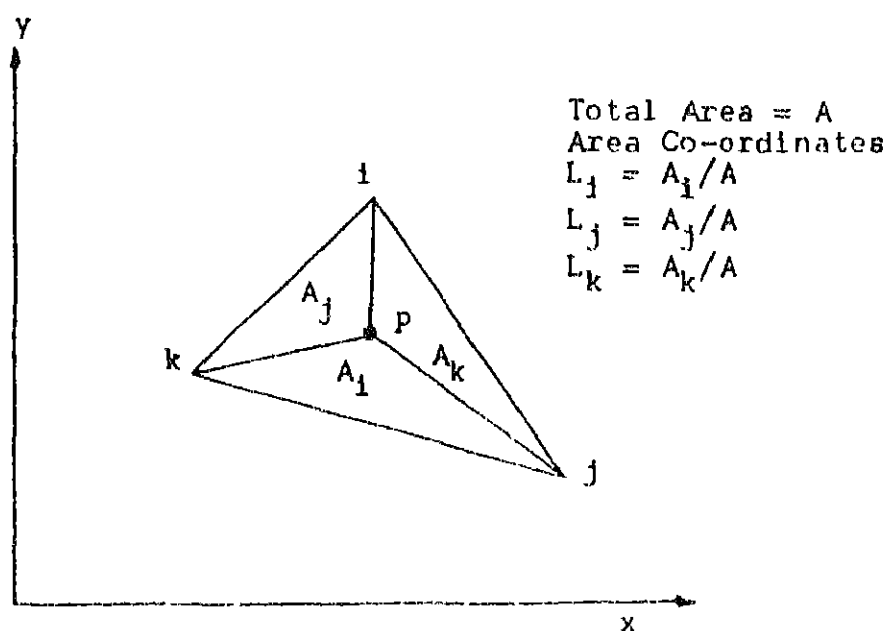


FIG. 3.1b AREA COORDINATES REPRESENTED BY  $L_i = A_i/A$

the element size shrinks to zero.

Considering any element (e) in the region D (Fig. 3.1 a) the function  $\phi$  within the element can be represented in terms of the nodal values as

$$\{\phi^{(e)}\} = [N^{(e)}] \{\phi\}^{(ne)} \quad (3.9)$$

where  $\{\phi^{(e)}\}$  is the variable within each element and  $\{\phi\}^{(ne)}$  represents a column matrix of the nodal values of the function and  $[N^{(e)}]$  is the row vector of interpolation functions, sometimes known as shape functions also, which are functions of the coordinates of the nodes. The function defining the state variable within each element must satisfy the compatibility and completeness criteria as outlined above.

The development of natural co-ordinates for triangular elements ( Fig. 3.1 b ) follow the choice of the co-ordinates  $L_i, L_j, L_k$ , also known as area co-ordinates, to describe the location of any point  $P(x_p, y_p)$  within the element or on its boundary as given by equation(3.9).

The original cartesian co-ordinates of a point in the element should be linearly related to the new co-ordinates by the following equations.

$$x = L_i x_i + L_j x_j + L_k x_k \quad (3.10)$$

$$y = L_i y_i + L_j y_j + L_k y_k \quad (3.11)$$

A third condition, requiring that the weighting functions sum to unity, is

$$L_i + L_j + L_k = 1 \quad (3.12)$$

so that only two of the natural co-ordinates can be independent.

The point  $(x_p, y_p)$  within the triangle  $ijk$  divides the total area into three separate areas as shown in (Fig.3.1 b). The ratios of these areas to the total area define uniquely the position of  $P$  within the triangle. These area co-ordinates are:

$$\left. \begin{aligned} L_i &= A_i/A \\ L_j &= A_j/A \\ L_k &= A_k/A \end{aligned} \right\} \quad (3.13)$$

These area co-ordinates are related to the cartesian  $x$  and  $y$  co-ordinates by the following equations

$$\left. \begin{aligned} L_i &= (a_i + b_i x + c_i y) / 2 A \\ L_j &= (a_j + b_j x + c_j y) / 2 A \\ L_k &= (a_k + b_k x + c_k y) / 2 A \end{aligned} \right\} \quad (3.14)$$

where,

$$\begin{aligned} a_i &= x_j y_k - x_k y_j, \quad a_j = x_k y_i - x_i y_k, \quad a_k = x_i y_j - x_j y_i \\ b_i &= y_j - y_k, \quad b_j = y_k - y_i, \quad b_k = y_i - y_j \\ c_i &= x_k - x_j, \quad c_j = x_i - x_k, \quad c_k = x_j - x_i \end{aligned} \quad (3.15)$$



and

$$2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \quad (3.16a)$$

or equation (3.16) can be re-written as

$$2A = [(x_1 y_j - x_j y_1) + (x_j y_k - x_k y_j) + (x_k y_1 - x_i y_k)] \quad (3.16b)$$

The field variable  $\phi$  may be interpreted as a function of  $L_i$ ,  $L_j$ ,  $L_k$  instead of  $x, y$  and  $z$  as

$$\phi^{(e)} = L_i \phi_i + L_j \phi_j + L_k \phi_k \quad (3.17)$$

It may be noted that  $L_i$  equals 1 at node  $i$  and equals 0 at nodes  $j$  and  $k$  and varies linearly between these points.

Also,

$$\begin{aligned} \frac{\partial \phi^{(e)}}{\partial x} &= \frac{1}{2A} (b_i \phi_i + b_j \phi_j + b_k \phi_k) \\ \frac{\partial \phi^{(e)}}{\partial y} &= \frac{1}{2A} (c_i \phi_i + c_j \phi_j + c_k \phi_k) \end{aligned} \quad (3.18)$$

The function  $\phi^{(e)}$  is thus uniquely and continuously defined throughout the region in the terms of nodal  $\phi$  values and therefore also satisfies the compatibility and completeness requirements.

### 3.4 Formulation of the Governing Equation in Matrix form:

The governing differential equation for steady two-dimensional seepage flow, as mentioned earlier is

$$\frac{\partial}{\partial x} (k_x \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial \phi}{\partial y}) + \bar{Q} = 0 \quad (3.19)$$

with the boundary conditions

$$\phi = \phi(x, y) \quad \text{on } S_1, \quad \text{and}$$

$$k_x \frac{\partial \phi}{\partial x} n_x + k_y \frac{\partial \phi}{\partial y} n_y + q = 0 \quad \text{on } S_2 \quad \text{with the usual notations.}$$

For the location of phreatic surface, the conditions on the phreatic surface that are to be satisfied are

$$i) \quad \phi = h \quad (\text{i.e. } p=0) \quad \text{where } h = \text{elevation head}$$

and ii)  $\frac{\partial \phi}{\partial n} = 0$  i.e. the flow across the phreatic surface is zero.

of (i) and (ii) only one of them is used as a boundary condition to obtain the solution of phreatic surface from the FE equations and the other is used as a check of the solution. Depending upon this check the iterations are performed on the equation.

If  $\phi$  is represented as equation (3.9) over an element, then the residual from equation (3.19), is

$$R^{(e)} = \frac{\partial}{\partial x} (k_x \frac{\partial \phi^{(e)}}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial \phi^{(e)}}{\partial y}) + \bar{Q} \quad (3.20)$$

According to Galerkin's criterion,

$$\iint_{D^{(e)}} N_1 R^{(e)} dx dy = 0 \quad (3.21)$$

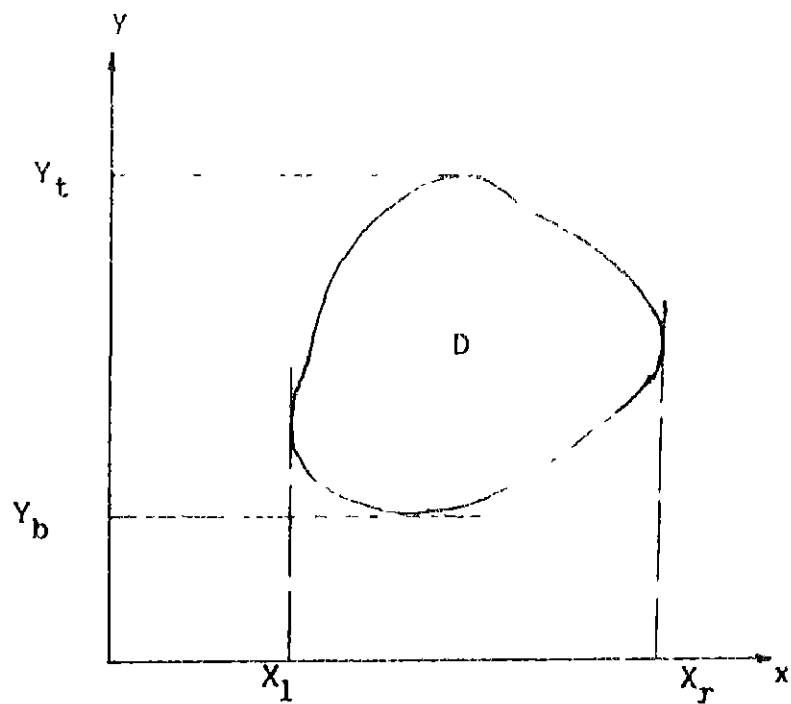


FIG. 3.2 EXTREME BOUNDARIES

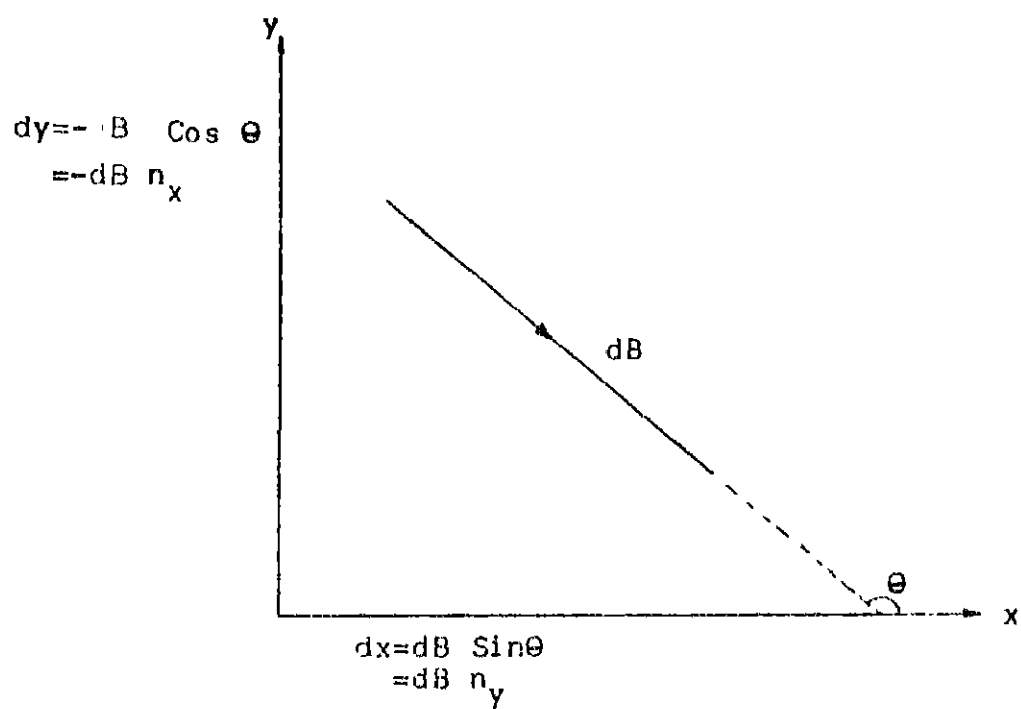


FIG. 3.3 DIRECTIONAL COMPONENTS

Substituting for  $R^{(e)}$  in the above, we get

$$\iint_{D^{(e)}} N_1 \left[ \frac{\partial}{\partial x} \left( k_x \frac{\partial \phi^{(e)}}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial \phi^{(e)}}{\partial y} \right) + \bar{Q} \right] dx dy = 0 \quad (3.22)$$

Let  $X_1$ ,  $X_r$ ,  $Y_b$ , and  $Y_t$  represent the extreme boundaries of the domain  $D$ , in the  $x$  and  $y$  directions respectively as shown in Fig. 3.2.

On integration by parts, Eq. (3.22) becomes

$$\begin{aligned} & \iint_{D^{(e)}} N_1 \left[ \frac{\partial}{\partial x} \left( k_x \frac{\partial \phi^{(e)}}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial \phi^{(e)}}{\partial y} \right) + \bar{Q} \right] dx dy \\ &= \int_{Y_b}^{Y_t} \left( N_1 k_x \frac{\partial \phi^{(e)}}{\partial x} \right) \Big|_{X_1}^{X_r} dy - \iint_{D^{(e)}} N_{1,x} k_x \frac{\partial \phi^{(e)}}{\partial x} dx dy \\ &+ \int_{X_1}^{X_r} \left( N_1 k_y \frac{\partial \phi^{(e)}}{\partial y} \right) \Big|_{Y_b}^{Y_t} dx - \iint_{D^{(e)}} N_{1,y} k_y \frac{\partial \phi^{(e)}}{\partial y} dx dy \\ &+ \iint_{D^{(e)}} N_1 \bar{Q} dx dy \end{aligned} \quad (3.23)$$

From Fig. 3.3, we have  $dy = -dB n_x$   
and  $dx = dB n_y$

Applying Stokes' transformation,

$$\int_{Y_b}^{Y_t} N_1 k_x \frac{\partial \phi^{(e)}}{\partial x} \Big|_{X_1}^{X_r} dy = \int N_{1,B} k_x \frac{\partial \phi^{(e)}}{\partial x} n_x dB \quad (3.24)$$

and,

$$\int_{X_1}^{X_r} N_1 k_y \frac{\partial \phi^{(e)}}{\partial y} \Big|_{Y_b}^{Y_t} dx = \int N_{1,B} k_y \frac{\partial \phi^{(e)}}{\partial y} n_y dB \quad (3.25)$$

Substituting equations (3.24) and (3.25) in Eq. (3.23) and regrouping, we get

$$\begin{aligned} & \iint_{D(e)} (k_x N_{1,x} \frac{\partial \phi(e)}{\partial x} + k_y N_{1,y} \frac{\partial \phi(e)}{\partial y}) dx dy \\ &= \iint_{D(e)} \bar{Q} N_1 dx dy + \int_{\Gamma} N_{1,B} (k_x \frac{\partial \phi(e)}{\partial x} n_x \\ & \quad + k_y \frac{\partial \phi(e)}{\partial y} n_y) dB \end{aligned}$$

or

$$\begin{aligned} & \iint_{D(e)} (k_x [N,x] \frac{\partial \phi(e)}{\partial x} + k_y [N,y] \frac{\partial \phi(e)}{\partial y}) dx dy \\ &= \iint_{D(e)} \bar{Q} [N] dx dy + \int_{\Gamma} [N,B] (k_x \frac{\partial \phi(e)}{\partial x} n_x \\ & \quad + k_y \frac{\partial \phi(e)}{\partial y} n_y) dB \end{aligned} \quad (3.26)$$

For the triangular element,  $\phi(e)$  is presented as

$$\phi(e) = a + bx + cy \quad (3.27)$$

then

$$\frac{\partial \phi(e)}{\partial x} = b \quad (3.28)$$

$$\frac{\partial \phi(e)}{\partial y} = c \quad (3.29)$$

Equations (3.27) to (3.29) show that there is completeness of  $n^{\text{th}} - 1$  order and compatibility of  $n^{\text{th}}$  order.

$$\begin{aligned} & \iint_{D(e)} (k_x [N,x] [N,x] + k_y [N,y] [N,y]) dx dy \{ \phi \}^{(ne)} \\ &= \iint_{D(e)} \bar{Q} [N] dx dy - \int_{\Gamma} [N,B] q dB \end{aligned} \quad (3.30)$$



$$\text{or } [k_\phi] \cdot \phi^{(ne)} = \bar{Q} - \int q_{II,B} dB \quad (3.31)$$

where,

$$[k_\phi^{(e)}] = \iint_{D(e)} (k_x [I', x] [II, x] + k_y [N, y] [N, y]) dx dy$$

$$k_\phi^{(e)} = \frac{1}{4A(e)} \begin{bmatrix} k_x b_1 b_1 + k_y c_1 c_1 & k_x b_1 b_j + k_y c_1 c_j & k_x b_1 b_k + k_y c_1 c_k \\ k_x b_1 b_j + k_y c_1 c_j & k_x b_j b_j + k_y c_j c_j & k_x b_k b_j + k_y c_k c_j \\ k_x b_1 b_k + k_y c_1 c_k & k_x b_j b_k + k_y c_j c_k & k_x b_k b_k + k_y c_k c_k \end{bmatrix} \quad (3.32)$$

where,

$$b_i = y_j - y_k, \quad b_j = y_k - y_i, \quad b_k = y_i - y_j$$

$$c_1 = x_k - x_j, \quad c_j = x_i - x_k, \quad c_k = x_j - x_i$$

### 3.5 Boundary Conditions:

For a complete specification of any problem, certain boundary conditions must be given. There are generally two cases:

- (a) the value of  $\phi$  is specified, This presents no difficulties and the value of  $\phi$  is prescribed and kept fixed at the appropriate nodes. In the present study, the boundary of the region of analysis with the GWT, the boundary of unlined part of the channel and the bottom layer if fully permeable constitute this boundary condition ( Fig. 3.4).

- (b) the velocity normal to an exterior boundary is specified. When this is the case,

$$v_x n_x + v_y n_y + q = 0 \quad (3.33)$$

where  $n_x, n_y$  are the direction cosines of the outward normal to the boundary surface and  $q$  is the given velocity normal to the boundary.

The negative term,  $\int q \cdot \mathbf{N} \cdot \mathbf{B} / dB$ , on the right hand side of Eq. (3.31) manifests this boundary condition. If  $q = 0$ , this term vanishes and the boundary condition that the velocity vector is tangent to the external boundary is automatically satisfied by the formulation above.

### 3.6 Generation of Phreatic Surface:

Before the equation (3.31) is finally solved, using the Gauss elimination technique, the boundary conditions discussed above are applied to the global stiffness matrix that had been assembled for the complete domain. The solution of Eq. (3.31) gives the  $\phi$  at each node in the domain. Though Zienkiewicz [32] and others have reported that, within the numerical accuracy, the finite element method gives the correct value of  $\phi$  throughout the region for fixed boundaries, in the present case the upper boundary i.e. the phreatic surface is not fixed and therefore the boundary conditions on it may not be at once fulfilled. Therefore, either a trial and error method, reported by Finn [20] or an iterative method, as



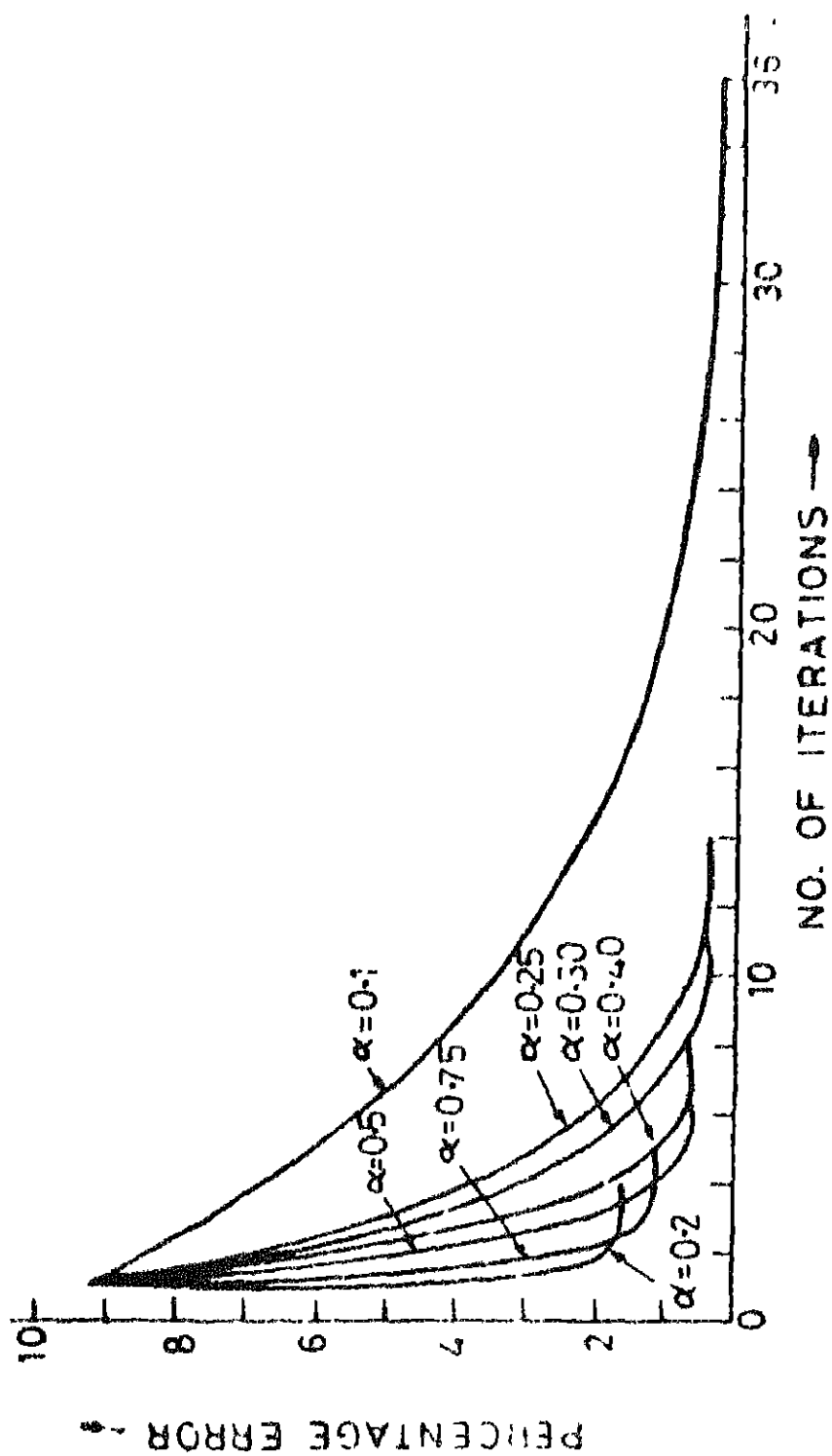


FIG.3.5 CONVERGENCE CRITERION

adopted in the present study, may be made use of.

In the iterative method an initial trial position is first chosen by the user and the FE analysis done. The results from that analysis can be used to generate automatically the next position of phreatic surface and the cycle repeated until a solution within numerical accuracy is achieved. It can be seen that this method is far less tedious on the part of the user. However, to automatically generate the phreatic surface nodes, the complete mesh too has to be regenerated. To this end, an automatic mesh generation technique was incorporated by Achar [23] in the program which was used for the present study after some modifications. The algorithm for automatic mesh regeneration is the same as given by Durocher and Gaspar [25]. The test of convergence for various relaxation factors ( $\alpha$ ) was carried out by Achar[23] . The result is reproduced in Fig 3.5 and has been adopted without change. Relaxation factor ( $\alpha$ ) is the factor applied to the difference between the  $\phi$  of the phreatic surface node obtained after FE analysis and the y-coordinate of that node on the mesh. The new position of the node is obtained by changing the old value by the product obtained above

$$\text{or } Y_{y_{i+1}} = y_i + \alpha \cdot \delta ; \text{ where,} \quad (3.34)$$

$\alpha$ = Relaxation factor

$$\delta = \phi_1 - y_1$$

### 3.7 Calculation of Velocity and Discharge:

Though emphasis is more on obtaining  $\phi$  (i.e. piezometric head) at various points in a domain using the FE technique, for the present study the velocity of flow and the actual quantity of discharge (or seepage) across a boundary are more important points of interest. The computation of these quantities from the primary variable at the node i.e.  $\phi$  is as follows:

By Darcy's law,

$$v = -k \nabla \phi \quad (3.35)$$

writing Eq. (3.35) in the matrix notations, for a two dimensional flow,

$$\begin{Bmatrix} v_x \\ v_y \end{Bmatrix} = - \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{Bmatrix} \partial\phi/\partial x \\ \partial\phi/\partial y \end{Bmatrix} \quad (3.36)$$

But the gradient matrix is written as

$$\begin{Bmatrix} \partial\phi/\partial x \\ \partial\phi/\partial y \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} b_1 & b_j & b_k \\ c_1 & c_j & c_k \end{bmatrix} \begin{Bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{Bmatrix} \quad (3.37)$$

where  $i, j, k$  represent the three nodes of the triangular element and  $\phi_i$  etc. the heads at node  $i$  etc.  $A$  is the area of the element.

Substituting equation (3.37) in equation (3.36), we get

$$\{v\} = \begin{Bmatrix} v_x \\ v_y \end{Bmatrix} = -\frac{1}{2A} \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{bmatrix} b_1 & b_j & b_k \\ c_1 & c_j & c_k \end{bmatrix} \begin{Bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{Bmatrix} \quad (3.38)$$

The quantity of flow is given by

$$\begin{aligned} Q &= vA \\ &= k \cdot 1 \cdot A \end{aligned} \quad (3.39)$$

The quantity of flow can also be related to the seepage velocities by computing equivalent flows at the nodes. For this purpose, the net flows in the x and y directions at the nodes are evaluated as the uniform flow tributary to the node and we get

$$\{Q\} = \begin{Bmatrix} Q_{xi} \\ Q_{xj} \\ Q_{xk} \\ Q_{yi} \\ Q_{yj} \\ Q_{yk} \end{Bmatrix} = -\frac{1}{2} \begin{bmatrix} b_1 & 0 \\ b_j & 0 \\ b_k & 0 \\ 0 & c_1 \\ 0 & c_j \\ 0 & c_k \end{bmatrix} \{v\} \quad (3.40)$$

where  $b_i, b_j, b_k$  and  $c_i, c_j, c_k$  are constants for a particular element and  $\{v\}$  is as calculated in Eq. (3.37).

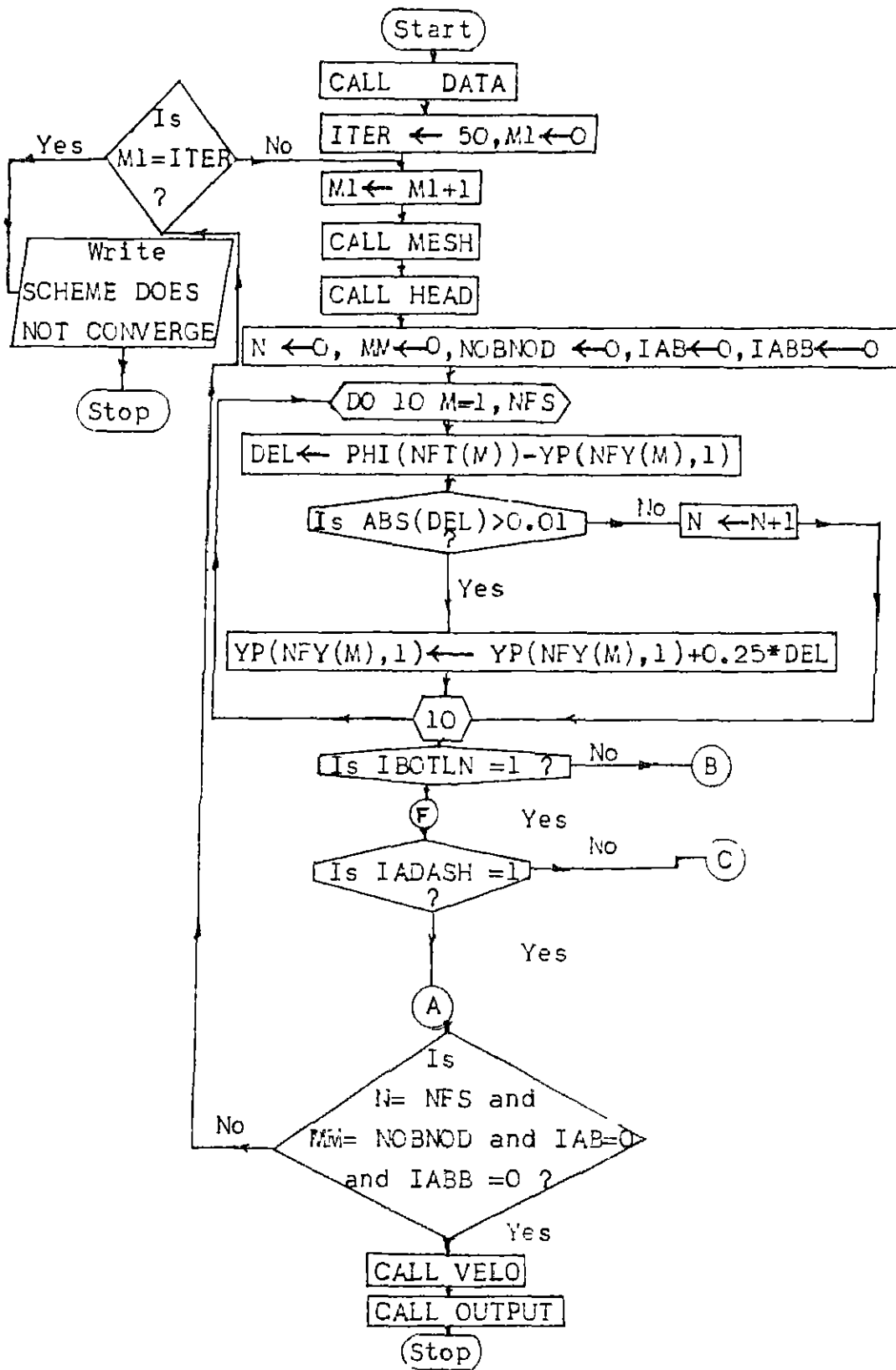


FIG. 4.1 FLOW CHART FOR MAIN PROGRAM

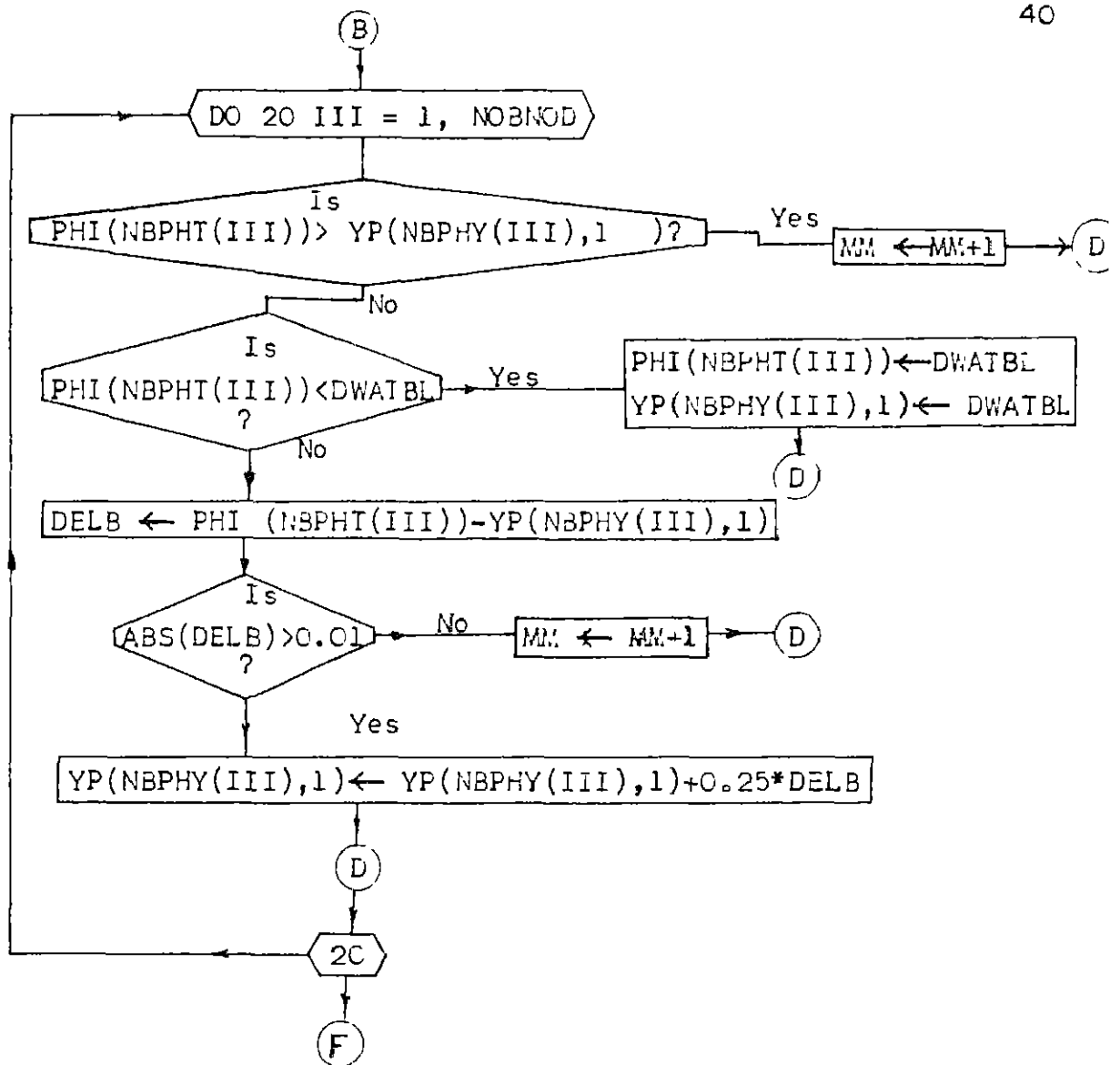


FIG. 4.2 FLOW CHART FOR BASE LINED CASE

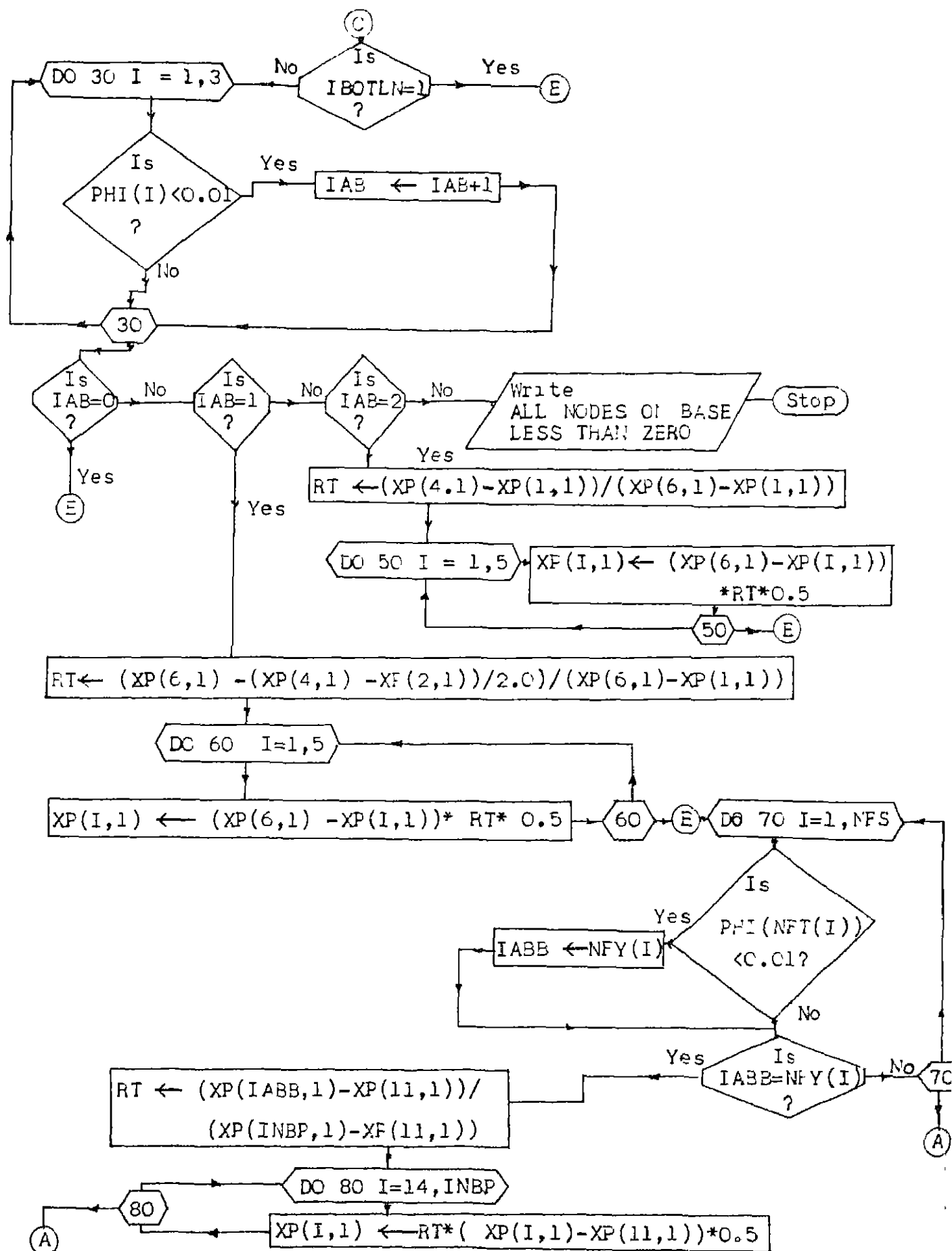


FIG. 4.3 FLOW CHART FOR A' CONDITION

## CHAPTER 4

### THE PROGRAM

The computer program 'CANSEP-86' used for the present study was developed by Achar [23]. It is a versatile program for dealing with the two dimensional steady state groundwater flow. Non-homogeneity and anisotropy can be easily handled. For the purpose of present study, a modification was made to incorporate the condition of seepage through channels lined at base only in which case there is a possibility of a bottom phreatic surface being generated. Also, to deal with A' condition (Fig. 2.1) of seepage where the phreatic surface meets orthogonally the underlying permeable layer certain changes were introduced. The phreatic surface in such a condition requires not only vertical adjustment so that the head of the node on the phreatic line is equal to the  $y$  co-ordinate of the node, but also horizontal displacement so that the condition of orthogonality is fulfilled. The flow chart for the main program is given in Fig. 4.1. The flow charts of the changes made are given in Fig. 4.2 and Fig. 4.3. The actual method of giving data input is explained in Appendix I with two examples.

#### 4.1 Important Subroutines:

The program consists of twelve subroutines out of which eight subroutines are actually used for the present study.



Following is the brief description of the various subroutines used.

1) Subroutine DATA:

This is the most important subroutine as it reads the complete specification of the problem, any error in giving the input can either lead to spurious results or abortion of the execution of the program. The geometry of the region of analysis, the various boundary conditions, material properties, the total number of nodes and elements, number of free surface nodes, etc. are all required to be specified. This is the only subroutine in the complete program that reads the input data, therefore any changes to be made in the style or content of input may be made in this subroutine only without searching the whole program for READ statements.

ii) Subroutine MESH:

The user specifies only the co-ordinates of the nodes of the superelements into which the region of analysis is initially divided. This subroutine subdivides the super-elements into specified number of nodes, number them and codifies their x and y co-ordinates. The actual formulation of elements from these nodes is done by subroutine LINEAR. Since this subroutine is derived from the work presented by Durocher and Gasper [25], a brief description of Automatic Mesh Generation Technique is given below: —

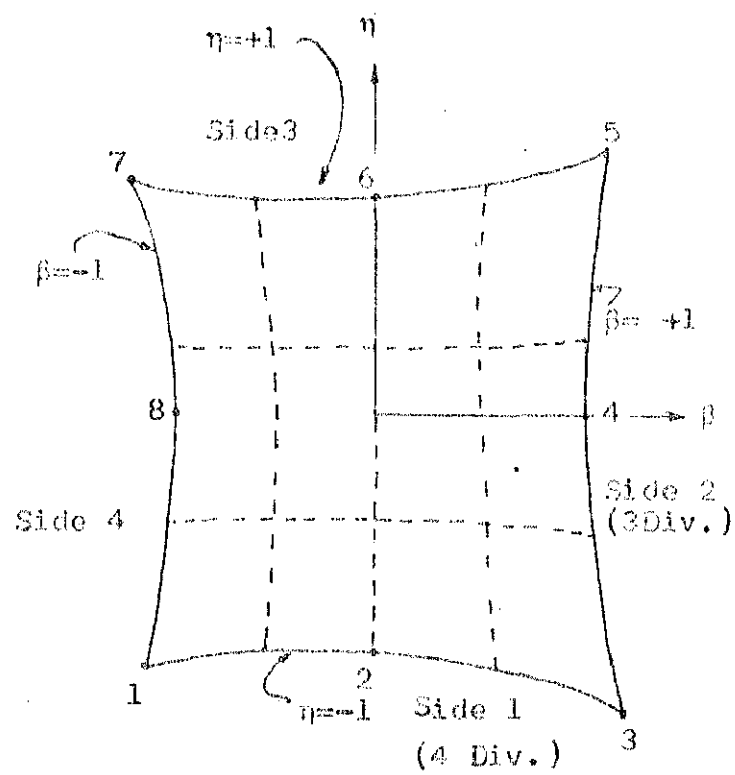


FIG. 4.4 TYPICAL PARABOLIC SUPER ELEMENT.

Zienkiewicz and Phillips have described the theory and technique employed in a mesh generator based on isoparametric mapping concepts. A typical parabolic super-element is shown in Fig.4.4, the  $x$  and  $y$  co-ordinates of a typical point within the superelement is related to the eight pairs of nodal co-ordinates by the equations:

$$x = \sum_{i=1}^n N_i (\beta, \eta) x_i \quad (4.1)$$

$$y = \sum_{i=1}^n N_i (\beta, \eta) y_i \quad (4.2)$$

where,  $N_i (\beta, \eta)$  is the shape function associated with node 'i'. The shape functions for parabolic quadrilaterals as well as other element types have been given by Zienkiewicz [32]. Thus, if the nodal co-ordinates  $(x_i, y_i)$  are known, the cartesian co-ordinates of any specified point  $(\beta, \eta)$  can be easily calculated from Eqs. (4.1) and (4.2). In case of the three noded triangle used for the present study, the eight noded superelement shown in Fig. 4.4 suffices. The relatively simple programming logic can be used to generate a mesh of a given refinement once the user specifies the number of element subdivisions in the  $\beta$  and  $\eta$  directions.

### iii) Subroutine LINEAR:

This subroutine divides the noded region of a superelement into finite elements which are triangles in the present study. It takes into consideration the fact that equilateral triangles produce more accurate results than

long narrow triangles by comparing the two diagonals of the quadrilateral produced by four nodes by subroutine MESH. It also generates the element connectivity array and material specification codes.

iv) Subroutine HEAD:

It generates the element stiffness matrix using the equations obtained in chapter 3. The element stiffness matrices are then assembled into the global stiffness matrix. By the help of subroutine SIMUL, the boundary conditions are applied to these equations. The subroutine SOLVE then solves by Gaussian elimination these simultaneous set of equations to obtain the nodal heads.

v) Subroutine SIMUL:

This subroutine introduces the boundary condition. The head wherever specified as the boundary condition is multiplied by the corresponding stiffness to obtain the nodal discharge. This discharge is then added to the right hand side of the equation reproduced below.

$$\{k\}^{(ne)} [\phi]^{(ne)} = [\bar{Q}]$$

Thus the total number of simultaneous equations is reduced to the number of nodes with specified  $\bar{Q}$  only and thus the set of equations can be solved to obtain  $\phi$  at different remaining nodes .

## vi) Subroutine SOLVE:

This subroutine solves the matrix equation obtained by subroutine SIMUL using the Gaussian elimination method.

## vii) Subroutine VELO:

This subroutine calculates the gradient ( $d\phi/ds$ ) over the element from which the velocity of flow over the element is obtained using Eqs. (3.37) and (3.38). From the elemental velocity the nodal discharges are calculated using equation (3.40). To obtain the total seepage from the channel the nodal discharges of the nodes lying on the boundary of the seepage zone are added up. For the present case, since the symmetry of the seepage zone is exploited by considering only half part of the zone for analysis, the total seepage quantity obtained by this subroutine is also half the actual seepage. Therefore, care should be taken when the total seepage quantity is required.

## viii) Subroutine OUTPUT:

This subroutine is called after the iterations have been completed and the phreatic surface(s) located. All the necessary information like element connectivity, heads at each node, nodal discharges, elemental velocities, co-ordinates of the phreatic surface, total seepage, etc. is printed out by it. Like the subroutine DATA, this also is self-contained and any change in the output may be made in it without

searching the whole program for WRITE statements.

#### 4.2 Some Physical Specifications of the Program:

i) The program can handle 200 nodes and 200 elements at a time. This limit is only due to the dimension allotted to the array of the nodes and elements. Larger numbers can be accommodated by increasing the dimensions of the arrays.

ii) The mesh generation program limited to the main program can generate a wide variety of elements ranging from 3-noded linear triangles to 8-noded isoparametric quadrilaterals. However, it is important to note that the program can be used for analysis by using 3-noded linear triangles only because the subroutine HEAD incorporates the stiffness matrix based on the interpolating function for linear triangles only. Therefore, necessary changes in subroutine HEAD will have to be made to accommodate higher and complex elements.

iii) The total CPU time needed to obtain the phreatic surface depends largely on the initial position estimated by the user. Thus, a proper engineering judgement must be exercised at the time of specifying the phreatic surface initially so that it is as close as possible to the actual position. However, it has been observed during the present study that the convergence becomes increasingly difficult with deeper GWT and also when GWT is at or below

the underlying permeable layer and CPU time as high as two minutes with approximately ninety iterations was required at times in such cases. The average CPU time was 30 seconds.

## CHAPTER 5

## SEEPAGE FROM PARTIALLY LINED CHANNELS

5.1 Introduction:

Using the FE program 'CANSEP-86' described earlier, the characteristics of seepage from unlined and partially lined channels under different boundary conditions were studied. The details of this investigation and the salient characteristics of seepage from partially lined channels are described in this chapter.

5.2 Basic Flow Conditions:

In actuality, seepage flow systems are characterized by channels of irregular cross section, non-uniformity of soils in horizontal as well as vertical extent, changing elevations of the water surface in the channel and the water table in the soil, and other complications. Perfectly impermeable lining of channel sides or bottom is an exception rather than the rule. Therefore, in view of the multitude of complexities with which the seepage flows are afflicted in nature, theoretical treatment must begin with the approximation of the soil and boundary conditions. The accuracy of the seepage and seepage reduction predicted for a given channel in that way then depends largely on how well the pertinent soil, water table, and boundary conditions can be characterized. Although



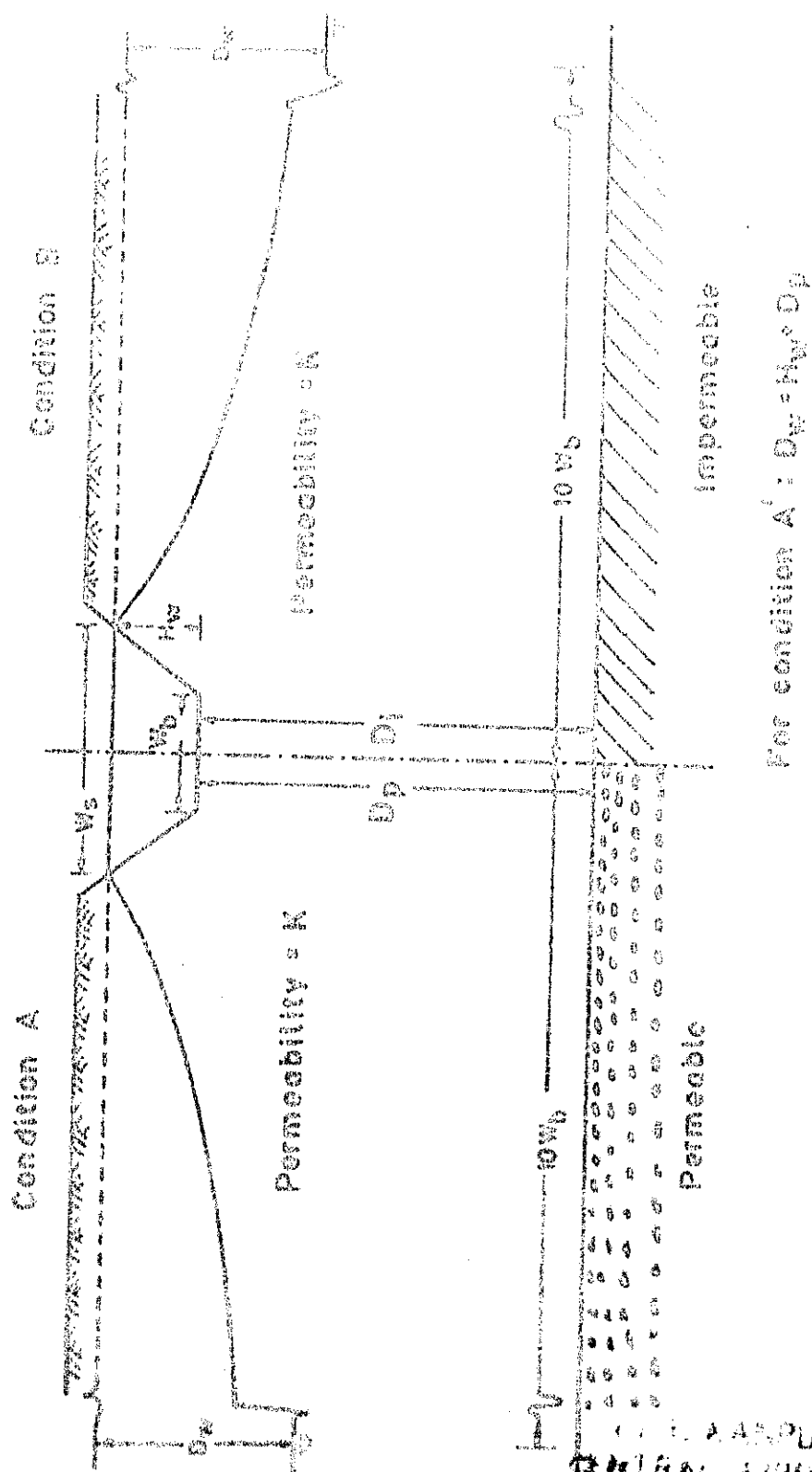


FIG. 5-1 DEFINITION SKETCH FOR SEEPAGE CONDITIONS A, B AND A'

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the FE analysis used for the present study yields accurate answers, the fact that the model for which the solution is obtained is always a simplified version of the field situation, renders the solution an estimate at best.

#### 5.2.1 Theoretical Models:

There are two basic conditions to which the multitude of natural profiles of soil hydraulic conductivity can be reduced for theoretical treatment of seepage flow systems. These conditions labeled A and B in accordance with Bouwer [11,12], are as follows : ( Refer Fig. 5.1)

Condition A: The soil in which the channel is laid is uniform and underlain by more permeable (considered here as infinitely permeable) material.

Condition B : The soil in which the channel is imbedded is uniform and underlain by less permeable (considered as impermeable) material.

The case of seepage to a free-draining permeable layer in the subsoil is a special case of condition A, and it is obtained by letting the water table be at or below the top of the permeable material. This condition is labeled A'.

#### 5.2.2 Significant Parameters Governing Seepage:

To obtain general solution, the depth of infinitely permeable ( $D_p$ ) or impermeable ( $D_i$ ) material for condition A and B respectively is treated as a variable. The position

TABLE 5.1

Range of Parameters Used in the Present Study

Variable	Description	Parameter	Range
$H_w$	Head of Water in the channel	$H_w/W_b$	0.25, 0.50, 1.0, 2.0
$D_w$	Position of water table from the water surface in the channel	$D_w/W_b$	1.0 to 3.0 depending upon $H_w/W_b$
$D_1$	Position of the underlying impermeable layer from the base of the channel	$D_1/W_b$	1.0, 2.0, 3.0
$D_p$	Position of the underlying infinitely permeable layer from the base of the channel	$D_p/W_b$	1.0, 2.0, 3.0
$m$	Side slope of the channel	$m$	1.0

of the water table at some distance from the channel is also treated as a variable ( $D_w$ ). This water table acts as a solid boundary for the present case of steady state seepage. Other variable is the head of water ( $H_w$ ) in the channel. All these parameters are non-dimensionalized with reference to the width of the base ( $W_b$ ) of the channel. Thus, for a given channel (say a trapezoidal channel of side slope,  $m$ ) the seepage quantity  $Q$  is written as,

$Q = \text{fn } (H_w, D_w, D_1 \text{ or } D_p, W_b, W_s, k)$ . Refer Fig. 5.1 for the definitions of these variables. Non-dimensionalization yields

$$\frac{Q}{W_s, k} = \frac{I_s}{k} = \text{fn } \left[ \frac{H_w}{W_b}, \frac{D_w}{W_b}, \frac{D_1}{W_b} \text{ or } \frac{D_p}{W_b}, m \right] \quad (5.1)$$

The value of  $m$  is kept constant for the present study as unity. The value of  $k$  ( $=0.01$  m/s) is also kept constant. The non-dimensionalized parameters and their ranges considered for the present study are given in Table 5.1.

### 5.3 Description of Problem:

Thus having simplified the actual conditions of seepage into two basic theoretical models and defined the governing parameters and their ranges of variation, the problem for this study reduces to obtaining the absolute seepage quantity for each of the models for all the possible combinations of the different values that each parameter can take for the channel being unlined, base only lined and sides only lined cases. Having obtained the absolute seepages, the percentage

reduction in seepage due to base or sides lining of the channel is calculated as,

$$\text{Percentage reduction in seepage} = \frac{Q_{UL} - Q_L}{Q_{UL}} \times 100 \% \quad (5.2)$$

where,

$Q_{UL}$  = seepage from unlined channel

$Q_L$  = seepage from lined ( base or sides) channel.

For a given case of lining and a set of parametric conditions, the actual seepage is obtained in two stages. In the first stage, using the formulations describe in chapter 3, the phreatic surface is determined. And in the second stage, the seepage quantity is determined by summing up the nodal discharges across a given section.

#### 5.4 Determination of the Phreatic Surface:

In determining the phreatic surface, the heads of the nodes supposed to lie on the phreatic surface are compared with their corresponding y co-ordinates. This is to satisfy the boundary condition of zero pressure head on the phreatic surface of an unconfined ground water flow system. If the difference between the y co-ordinate and the head is greater than 0.01 meter then the y co-ordinate is modified using the expression

$$y_{\text{new}} = y_{\text{old}} + (\phi_{\text{node}} - y_{\text{old}}) \cdot \alpha \quad (5.3)$$

where ,

$y_{\text{new}}$  = modified y co-ordinate.

$y_{\text{old}}$  = previous y co-ordinate.

$\phi_{\text{node}}$  = head of the node on the phreatic surface.

$\alpha$  = relaxation factor.

The cycle of new mesh generation obtaining the head of all the nodes in the region of analysis and the comparison of heads and y co-ordinates of phreatic surface nodes and the modification of y co-ordinates of nodes lying on the phreatic surface using Eq. (4.3) is repeated until for all the nodes on the nodes on the phreatic surface the difference between the y co-ordinates and the heads of nodes on phreatic surface is less than 0.01 meter.

For the case when the bottom of the channel is also lined, there is a possibility of the generation of the bottom phreatic surface. In such condition, the solution is said to have been obtained only when the condition of difference between the heads of the nodes on the phreatic surface and their y co-ordinates being less than 0.01 meter is satisfied simultaneously for both the general phreatic surface and the bottom phreatic surface.

#### 5.4.1 Choice of the Relaxation Factor:

The choice of the relaxation factor governs the convergence of the problem. A poor choice may cause large

oscillations of the values obtained about the actual value. For the present study, the  $\alpha$  obtained by Achar [23] after conducting a test ( Fig. 3.5) was adopted without modification, after having been checked and found satisfactory keeping in view the accuracy achieved and the CPU time required. The  $\alpha$  adopted for the present study is 0.25. The same  $\alpha$  is seen to hold good for the bottom phreatic surface also.

### 5.5 Seepage Quantity:

The equations for obtaining the quantity of seepage in matrix form are given in the section 3.7 of chapter 3. The total quantity of seepage across the channel is obtained by summing up the nodal discharges in both x and y direction across any selected section. As for the present study the symmetry of channel geometry and surrounding soil and water table conditions is exploited thereby permitting analysis of only half of the region of seepage, the discharge so calculated is also half of the actual quantity of seepage taking place through the channel. Therefore, if absolute quantity of seepage through the channel is required, then the quantity obtained from the program 'CANSEP -86' should be multiplied by two.

### 5.6 Optimum Mesh Size:

The size of the mesh plays a vital role in the accuracy of the results obtained by the FE analysis. It is an

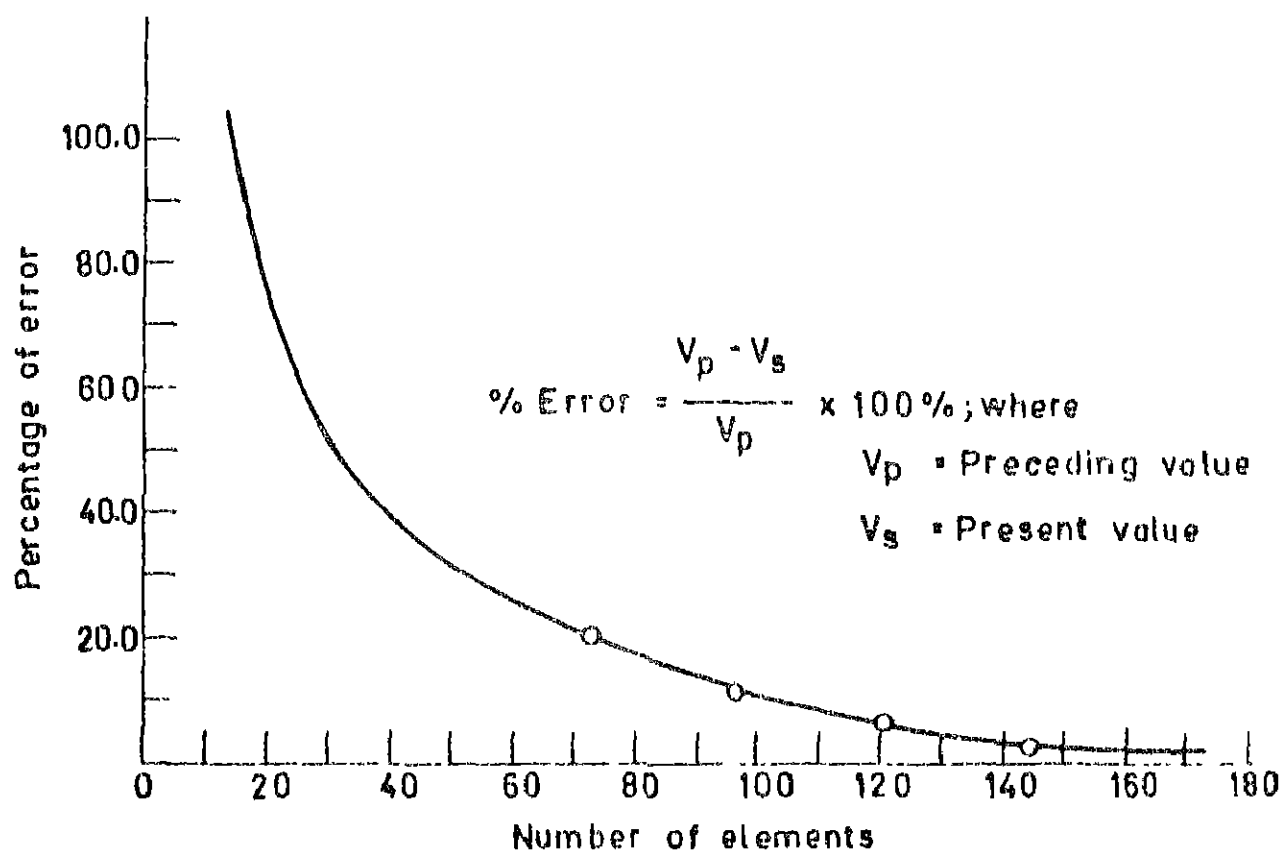


FIG 5.2 TEST FOR OPTIMUM NUMBER OF ELEMENTS



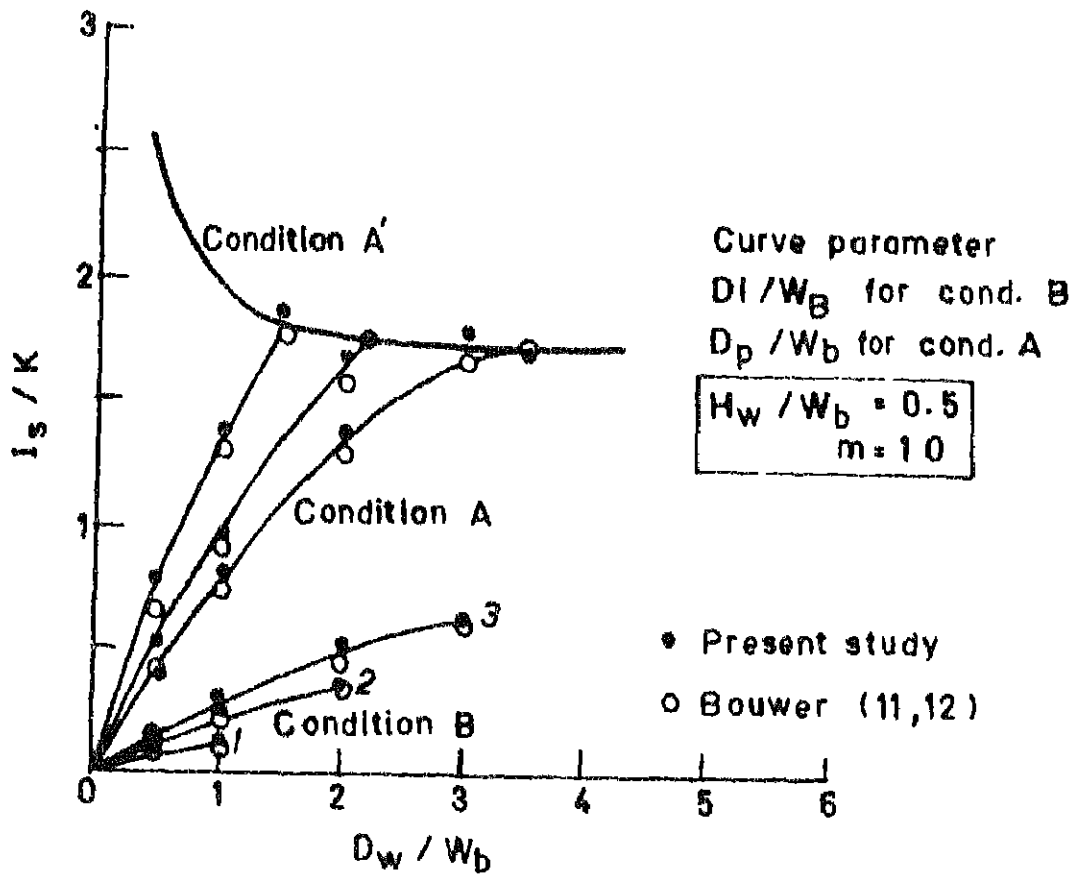


FIG 5.3 VERIFICATION OF RESULTS OF PRESENT STUDY FOR UNLINED CHANNEL WITH BOUWER'S (11,12) RESULTS

present study is not to stipulate precisely the amount of seepage reduction by lining the sides or the bottom of the channel but to provide a close estimate of the actual reduction in seepage that may take place.

### 5.7 Verification of the Program:

It was essential to establish the authenticity of the program before it could be put to further use by verifying the results produced by it with some established work done earlier. Bouwer [11,12] has presented a set of curves for seepage through unlined channels using electric analog method for various values of the parameter  $H_w/W_b$ . Bouwer had, in turn, verified his results with the results obtained by investigators like Dachler, Ernst, Vedernikov and Hammad [4]. Therefore, Bouwer's results can be assumed to be reliable and verification with them can be taken as the proof of the authenticity of the program used for the present study.

Fig. 5.3 shows a comparison of the values obtained by using the program and Bouwer's [11,12] values. These curves are for an unlined channel of side slope 1H:1V and  $H_w/W_b = 0.5$ . The variable  $D_w/W_b$  is plotted on the x-axis while the dependent variable  $I_s/k$  (Eq.5.1) is on the y-axis. The curve parameter is  $D_i/W_b$  or  $D_p/W_b$  for condition B and A respectively. The values are in good agreement with Bouwer's results. However, it may be noted that when the values of the variable  $D_w/W_b$  are such that the water table is quite close to the underlying

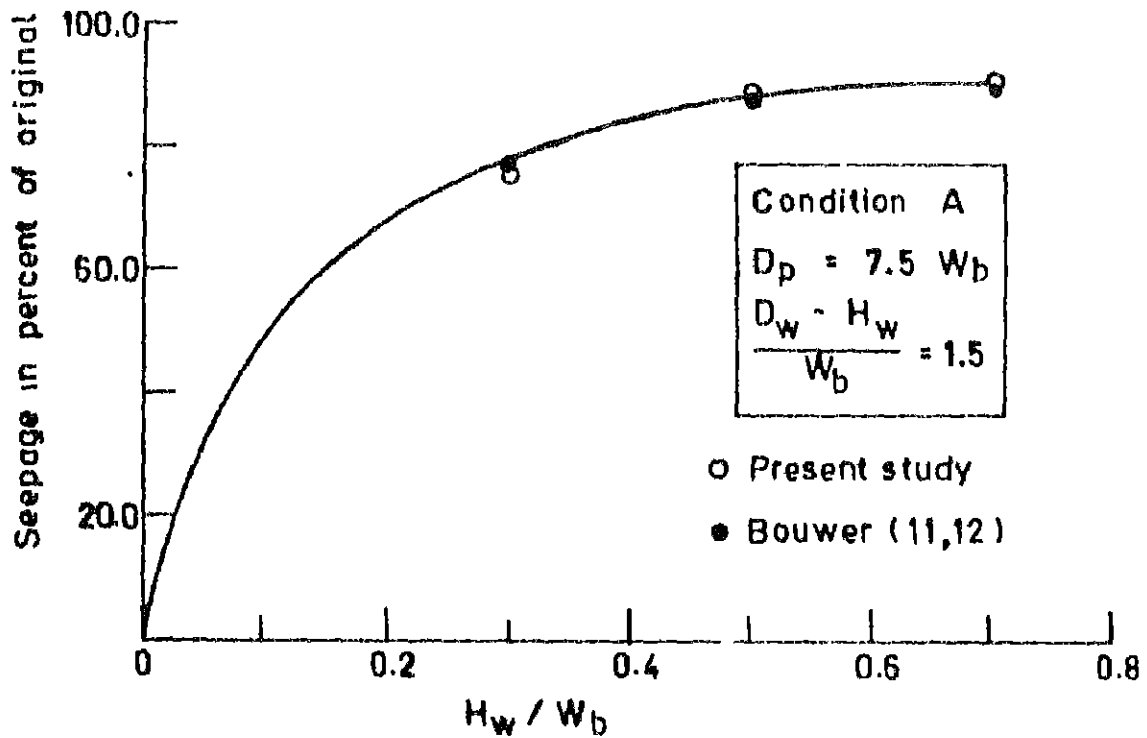


FIG. 54. VERIFICATION OF RESULTS OF PRESENT STUDY FOR CHANNEL LINED AT THE BOTTOM WITH BOUWER'S (11,12) RESULTS

bottom layer (Refer Fig. 5.1) the difference between Bouwer's results and the results obtained by the present study increases. The reason for this may be that when the position of the groundwater table (GWT) below the channel is quite deep the hydraulic gradient becomes steeper but as the mesh size remains constant for all positions of GWT the percentage error increases slightly above normal. The difference is of the order of about 3%.

Fig. 5.4 shows a comparison of the reduction in seepage obtained by lining the bottom of the channel when the underlying bottom layer is highly permeable (condition A) for the parameters indicated as obtained by Bouwer [11,12] and by using the program. Again, the results are in excellent agreement.

## 5.8 Seepage Through Partially Lined Channels:

The program 'CANSEP-86' was executed repeatedly for unlined, sides lined and base lined cases for a given set of parameters indicated in Table 5.1 to obtain the seepage quantity in each case from which the percentage reduction in seepage was obtained using Eq. 5.2. The conditions A and B were treated separately.

### 5.8.1 Seepage for Channels with Sides Lined and Base Unlined:

Seepages for the channel with sides lined for the complete range of  $D_i$  and  $D_w$  for condition B and of  $D_p$  and  $D_w$  for condition A were obtained. The side lining of the channel

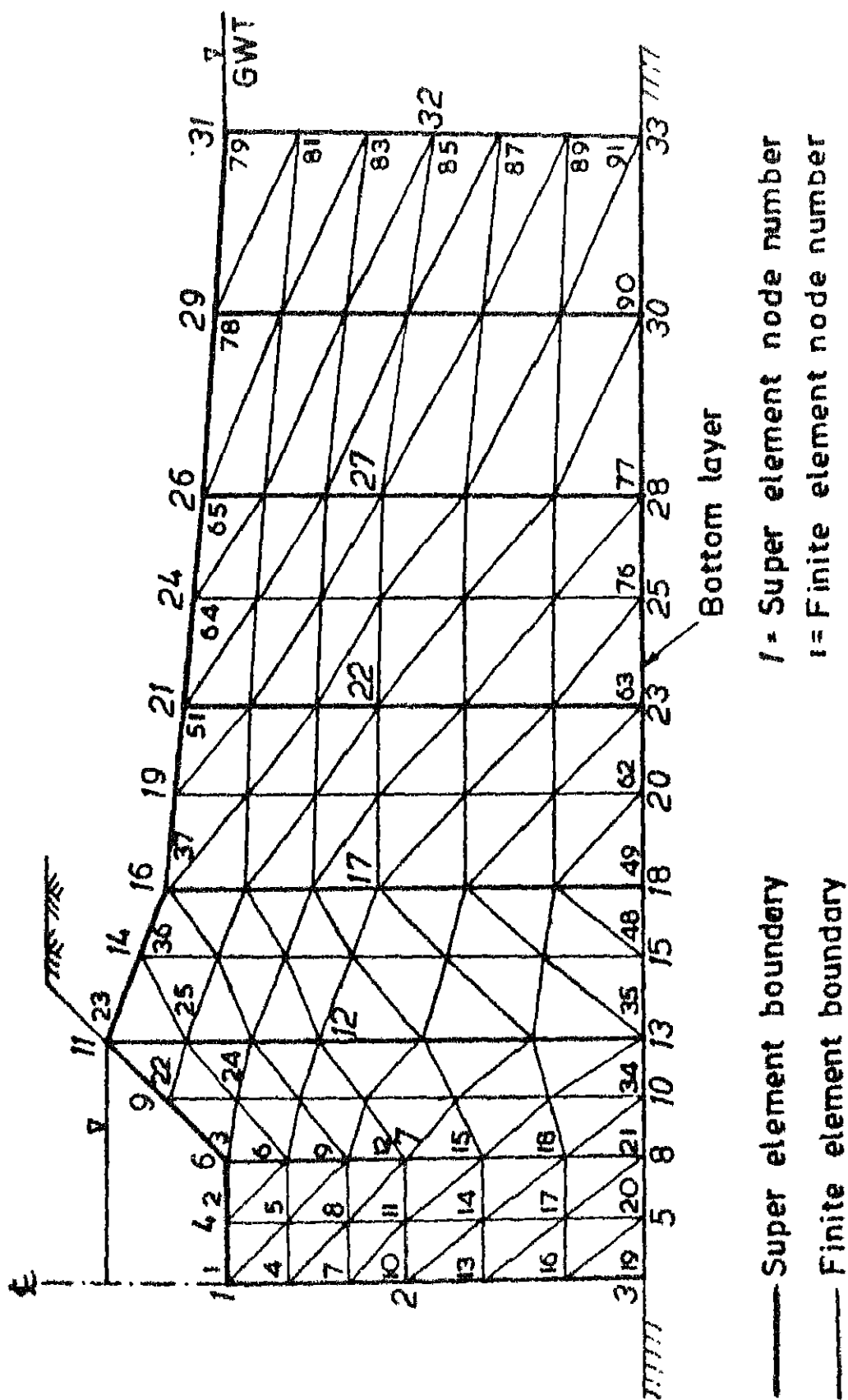


FIG. 5.5 TYPICAL MESH SHOWING SUPERELEMENTS AND  
FINITE ELEMENTS

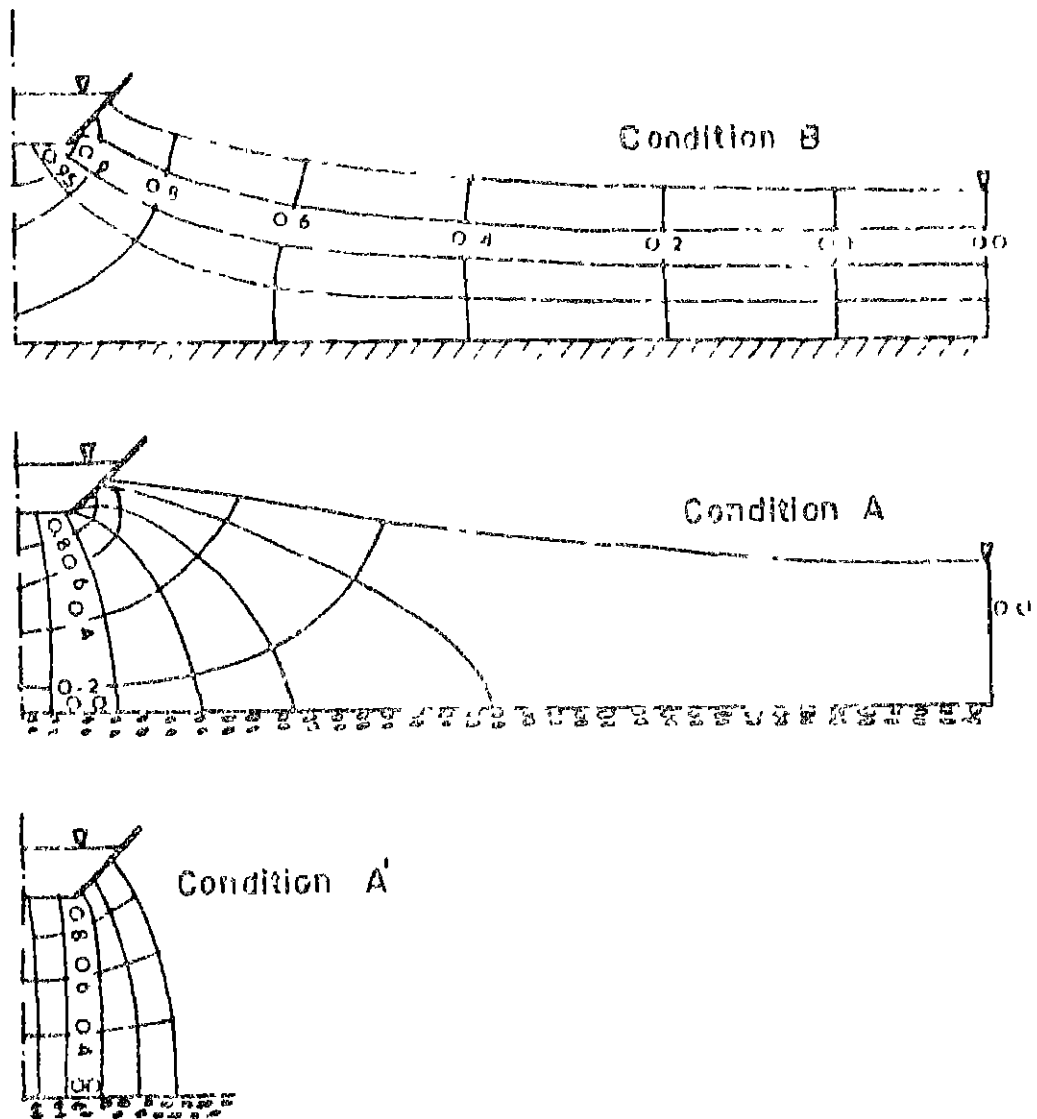


FIG 5.6 TYPICAL FLOW SYSTEMS OBTAINED FOR SEEPAGE THROUGH CHANNEL WITH SIDES LINED AND BOTTOM UNLINED

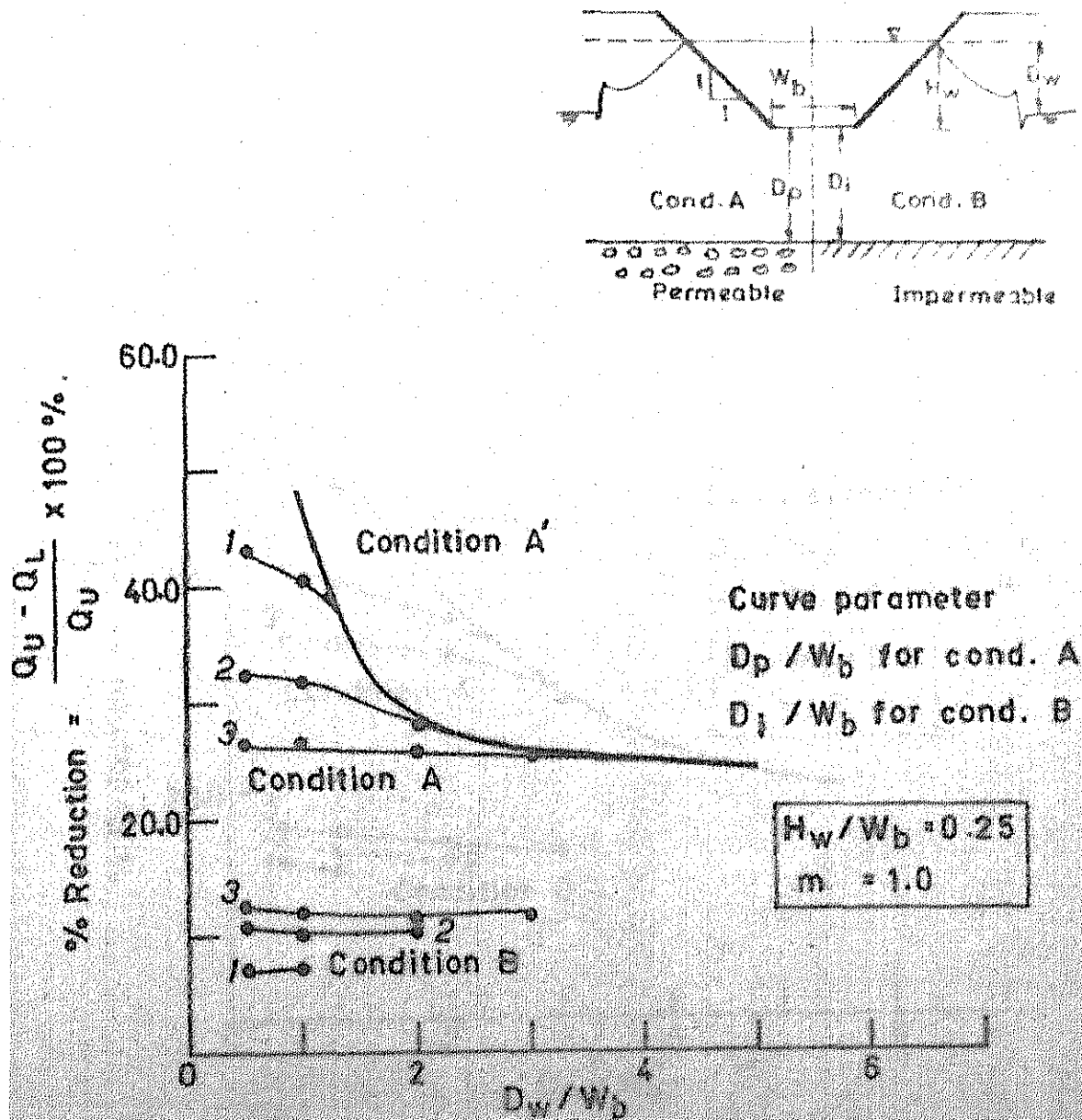


FIG. 5.7a. RESULTS OF SEEPAGE ANALYSIS FOR SIDES LINING OF CHANNEL WITH  $H_w / W_b = 0.25$  AND  $m = 1.0$

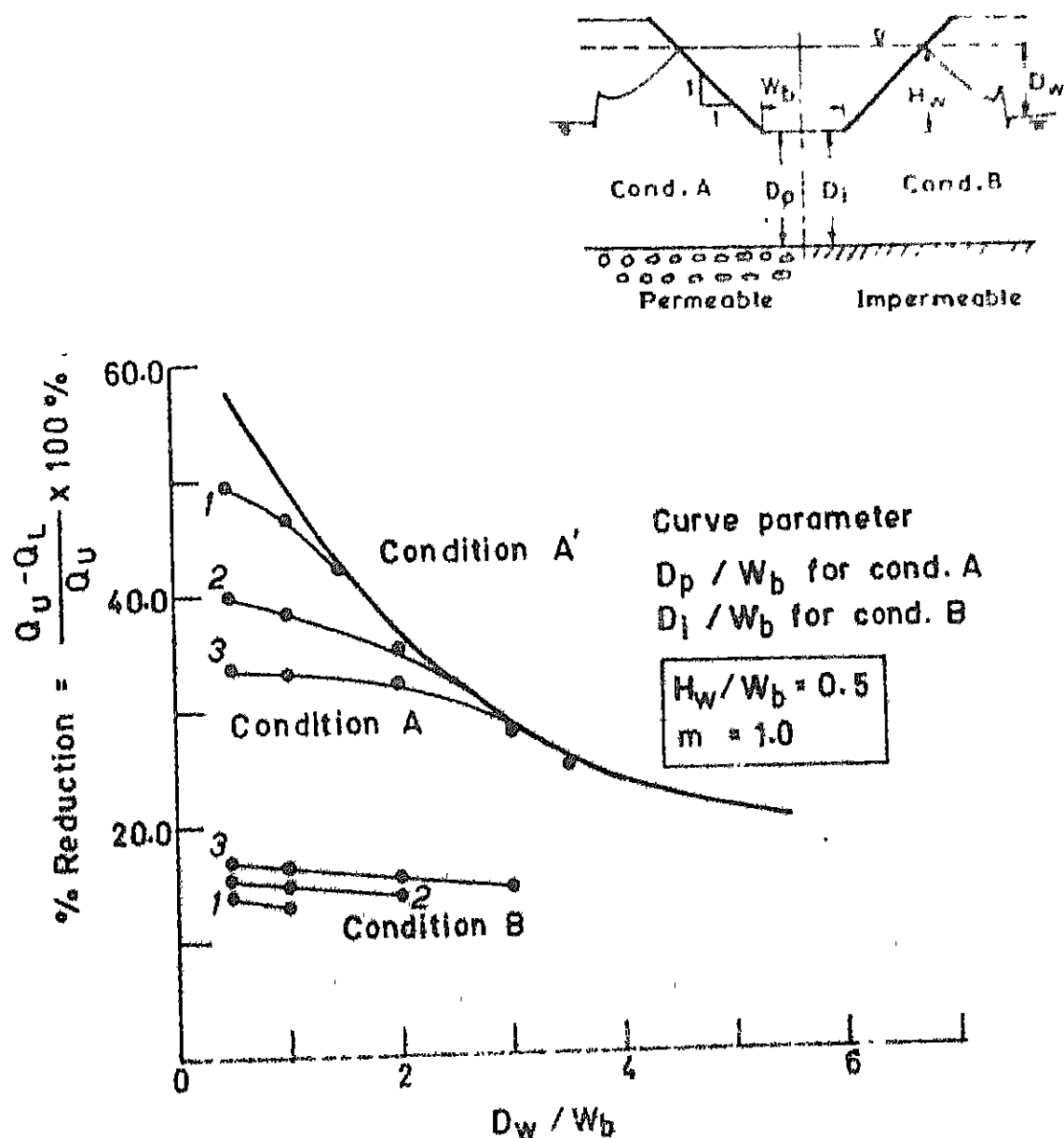


FIG. 57b. RESULTS OF SEEPAGE ANALYSIS FOR SIDES LINING OF CHANNEL WITH  $H_w / W_b = 0.5$  AND  $m = 1.0$



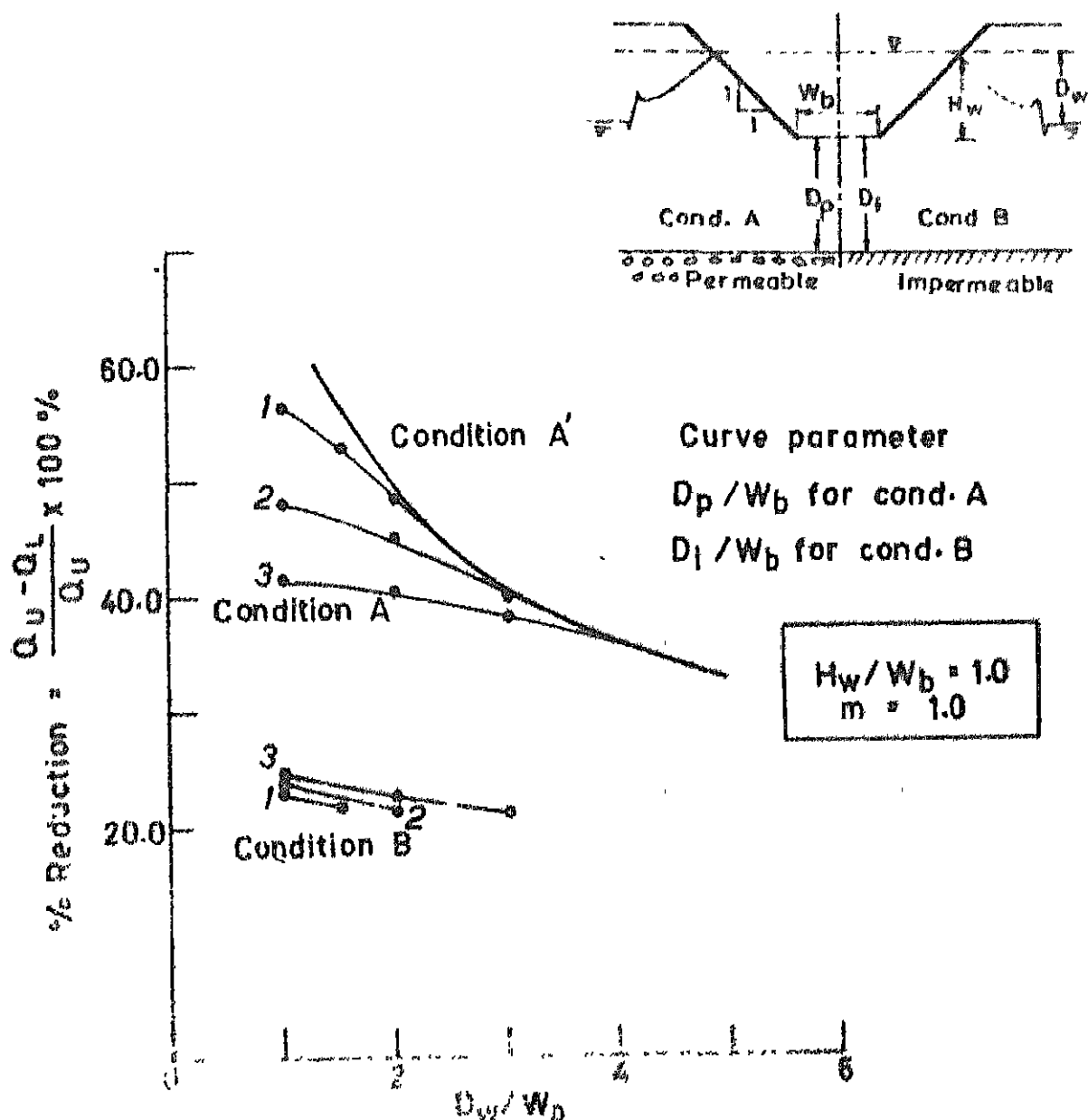


FIG. 3.7c RESULTS OF SEEPAGE ANALYSIS FOR SIDES LINING OF CHANNEL WITH  $H_w / W_b = 1.0$  and  $m = 1.0$

TABLE 5.2

Results of Seepage analysis for sides lined and base not lined case

$W_b = 3.0$  mts. for condition A.  $W_b = 6.0$  mts. for condition B  
(or as indicated)

$H_w/W_b = 0.25$  ( $W_b=2.0$  mts. for cond. B)

Results for Condition A				Results for Condition B					
$D_p/W_b$ or $D_i/W_b$	$D_w/W_b$	Seepage Qty. for UL channel el; $Q_L$ ( $m^3/s$ ) $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	Seepage Qty. for SL channel el; $Q_L$ ( $m^3/s$ ) $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	% Reduction	Lining Factor ( $f_L$ )	Seepage Qty. for UL channel el; $Q_L$ ( $m^3/s$ ) $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	Seepage Qty. for SL channel el; $Q_L$ ( $m^3/s$ ) $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	% Reduction	Lining Factor ( $f_L$ )
1.0	0.5	2.016	1.144	43.25	-0.046	0.113	0.105	6.93	-0.716
	1.0	3.716	2.213	40.45	-0.098	0.173	0.160	7.67	-0.703
	0.5	1.326	0.896	32.42	-0.246	0.210	0.187	10.93	-0.642
2.0	1.0	2.458	1.672	31.98	-0.254	0.373	0.335	10.22	-0.655
	2.0	4.248	3.056	28.05	-0.326	0.557	0.501	10.05	-0.641
	0.5	1.101	0.810	26.41	-0.357	0.290	0.254	12.45	-0.614
3.0	1.0	2.069	1.514	26.83	-0.350	0.531	0.468	11.87	-0.625
	2.0	3.437	2.547	25.89	-0.366	0.873	0.780	10.66	-0.647
	3.0	4.738	3.525	25.61	-0.372	1.048	0.949	10.03	-0.620
$H_w/W_b = 0.50$									
	0.5	2.308	1.166	49.48	-0.220	0.437	0.376	13.86	-1.080
1.0	1.0	4.245	2.269	46.55	-0.290	0.723	0.629	13.01	-1.100
	0.5	1.555	0.933	40.03	-0.448	0.740	0.624	15.67	-1.036
2.0	1.0	2.896	1.777	38.64	-0.481	1.337	1.137	14.95	-1.053
	2.0	5.011	3.234	35.46	-0.557	2.116	1.821	13.94	-1.077

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$H_w/W_b = 0.50$ (Contd....)									
3.0	0.5	1.279	0.847	33.82	-0.597	0.982	0.814	17.08	-1.002
	1.0	2.416	1.613	33.27	-0.611	1.836	1.529	16.72	-1.010
	2.0	4.125	2.788	32.41	-0.632	3.127	2.635	15.73	-1.034
	3.0	5.639	4.054	28.10	-0.735	3.764	3.220	14.45	-1.065
$H_w/W_b = 1.0$									
1.0	1.0	5.481	2.380	56.58	-0.663	1.088	0.838	22.95	-1.951
	1.5	6.793	3.225	52.53	-0.818	1.657	1.300	21.50	-2.005
	1.0	3.685	1.911	48.14	-0.986	1.719	1.321	23.15	-1.943
2.0	2.0	6.608	3.631	45.05	-1.104	2.805	2.215	21.03	-2.025
	1.0	2.987	1.744	41.61	-1.236	2.250	1.719	23.60	-1.926
3.0	2.0	5.444	3.237	40.54	-1.277	3.876	3.004	22.50	-1.968
	3.0	7.213	4.475	37.96	-1.376	4.951	3.902	21.19	-2.018
$H_w/W_b = 2.0$									
2.0	2.0	13.900	5.057	63.62	-1.423	3.137	1.964	37.39	-3.170
	2.5	15.746	6.448	59.07	-1.727	4.780	3.120	34.86	-3.330
	2.0	9.267	4.052	56.27	-1.912	4.447	2.940	33.68	-3.617
2.0	3.0	12.890	6.048	53.09	-2.125	5.688	3.865	32.05	-3.525
	2.0	7.492	3.703	50.58	-2.292	5.651	3.721	34.15	-3.385
3.0	3.0	10.470	5.415	48.28	-2.444	7.463	5.069	32.08	-3.524

For Condition A'							
$\frac{H_w}{W_b}$	$\frac{D_p}{W_b}$	$\frac{D_w}{W_b}$	Width of base $W_b$ (m)	Seepage Qty. for UL channel $Q_U$ (m <sup>3</sup> /s) $\times 10^{-2}$	Seepage Qty. for BL channel $Q_L$ (m <sup>3</sup> /s) $\times 10^{-2}$	Percentage reduction	Lining Factor ( $f_L$ )
0.25	1.0	1.25	2.0	2.400	1.440	39.08	-0.024
	2.0	2.25	2.0	2.387	1.729	27.58	-0.237
	3.0	3.25	2.0	2.321	1.810	25.68	-0.331
0.50	1.0	1.50	3.0	4.907	2.813	42.67	-0.384
	2.0	2.50	3.0	5.306	3.583	32.48	-0.630
	3.0	3.50	3.0	5.707	4.275	25.07	-0.809
1.0	1.0	2.00	3.0	9.065	4.668	48.50	-0.971
	2.0	3.00	3.0	7.426	4.447	40.12	-1.293
	3.0	4.00	3.0	7.250	4.664	35.67	-1.463
2.0	1.0	3.00	3.0	17.520	7.587	56.70	-1.883
	2.0	4.00	3.0	15.060	7.835	47.97	-2.463
	3.0	5.00	3.0	14.760	8.896	39.73	-3.012

was achieved by specifying zero nodal discharges at nodes 3, 22 and 23 of the FE mesh ( Fig. 5.5). The condition B was simulated by specifying zero nodal discharges at nodes lying on the bottom layer while condition A necessitated specifying the heads equal to the GWT at the nodes lying on the bottom layer. The position where the phreatic surface met the GWT was taken as ten times the width of the base of the channel [12]. Since the results are plotted in terms of percentage reduction in seepage with respect to the seepage through unlined channel the seepage quantity for the same set of parameters and conditions as for the side lined case was obtained for the unlined case. Typical flow systems for condition B, condition A and condition A' are shown in Fig. 5.6. The results of the analysis are presented in terms of dimensionless graphs showing percentage reduction as a function of  $D_w/W_b$  for different values of  $D_i/W_b$  (condition B) and  $D_p/W_b$  (condition A) (Fig. 5.7 a, b, c). The curve joining the end points of the curves obtained for condition A and different values of  $D_p/W_b$  represents the condition A' when the GWT lies at or below the underlying infinitely permeable layer. It may be noted for A' condition,  $D_w = D_p + H_w$ , any further lowering of water table does not increase the effective value of  $D_w$ . Thus, for condition A', the  $D_w$  values at the abscissa should be interpreted as  $D_p + H_w$ . The analysis was performed for trapezoidal channels with side slope of 1H:1V and four different water depths (expressed as  $H_w/W_b$ ). The values are presented in Table 5.2.

The curves, in Fig. 5.7, for percentage reduction in seepage for a channel with sides only lined show that for condition B there is very little effect of  $D_w$  on the percentage reduction of seepage. However, for condition A,  $D_w$  significantly affects percentage reduction in seepage but for  $D_w$  less than 6 times  $W_b$  only. Therefore, for condition A, the effect of water table deeper than 6 times  $W_b$  can be ignored without causing appreciable error in the percentage reduction in seepage obtained due to side lining of the channel.

The curves indicate the depth of impermeable layer from the base of the channel ( $D_1$ ) for condition B tends to increase the percentage reduction in seepage with increase in the  $D_1$ . However, this increase in percentage reduction in seepage is very less. For condition A, the depth of permeable layer from the base of the channel ( $D_p$ ) plays a significant role in the percentage reduction in seepage. The increase in  $D_p$  causes significant decrease in the percentage reduction in seepage. However, the curves for percentage reduction in seepage versus the  $D_w/W_b$  tend to come closer to each other as the  $D_p/W_b$  values increase. As can be seen in Fig. 5.7 a, the curve for  $D_p/W_b = 3$  is almost a straight line, indicating thereby that any further increase in  $D_p$  would not significantly affect the percentage reduction in seepage. Similar trends are apparent for curves in Fig. 5.7 b and Fig. 5.7 c.

The percentage reduction in seepage for a given set of conditions increases with the  $H_w/W_b$  values when the sides of

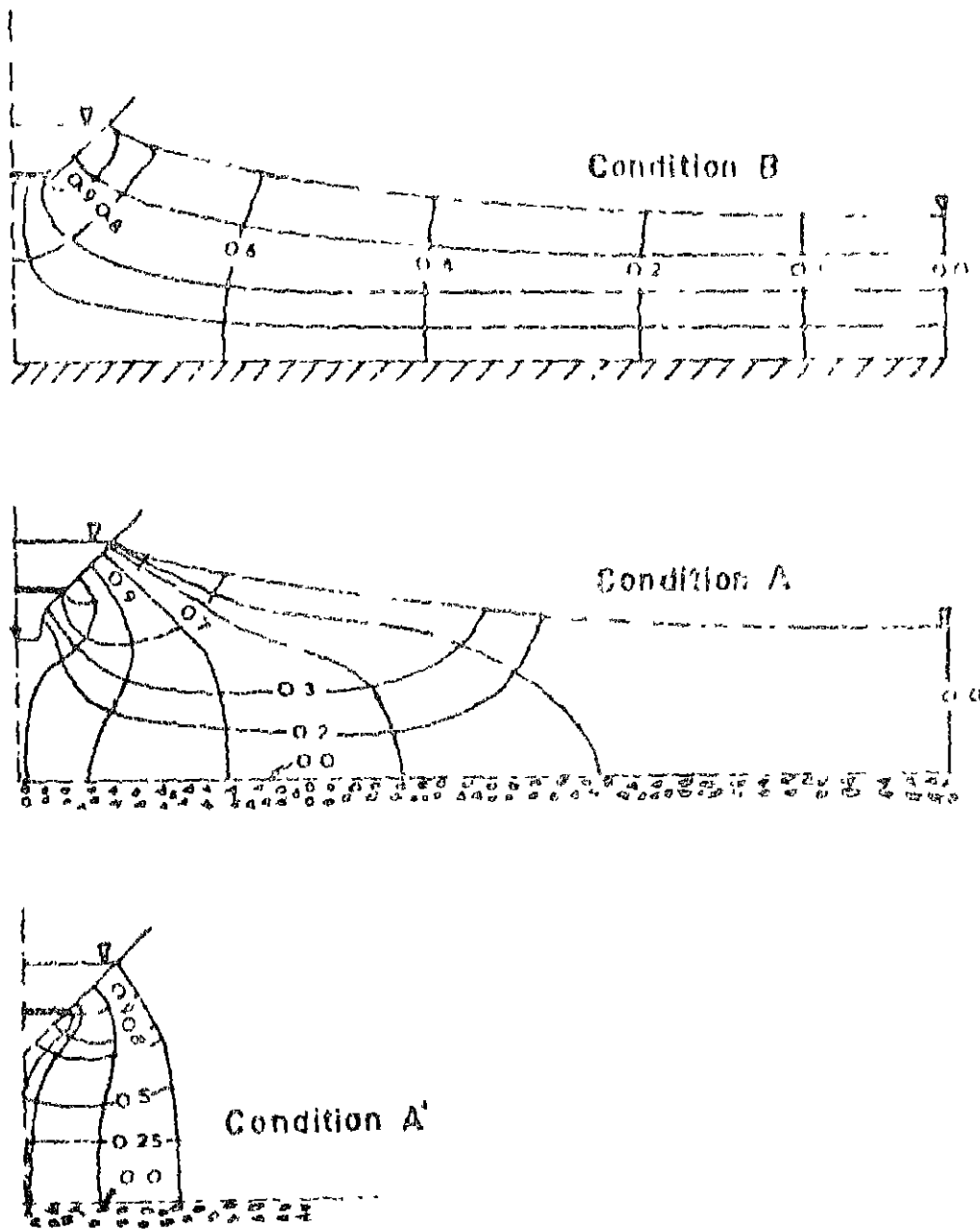


FIG. 5.8 TYPICAL FLOW SYSTEMS OBTAINED FOR SEEPAGE THROUGH CHANNEL WITH BASE LINED AND SIDES UNLINED

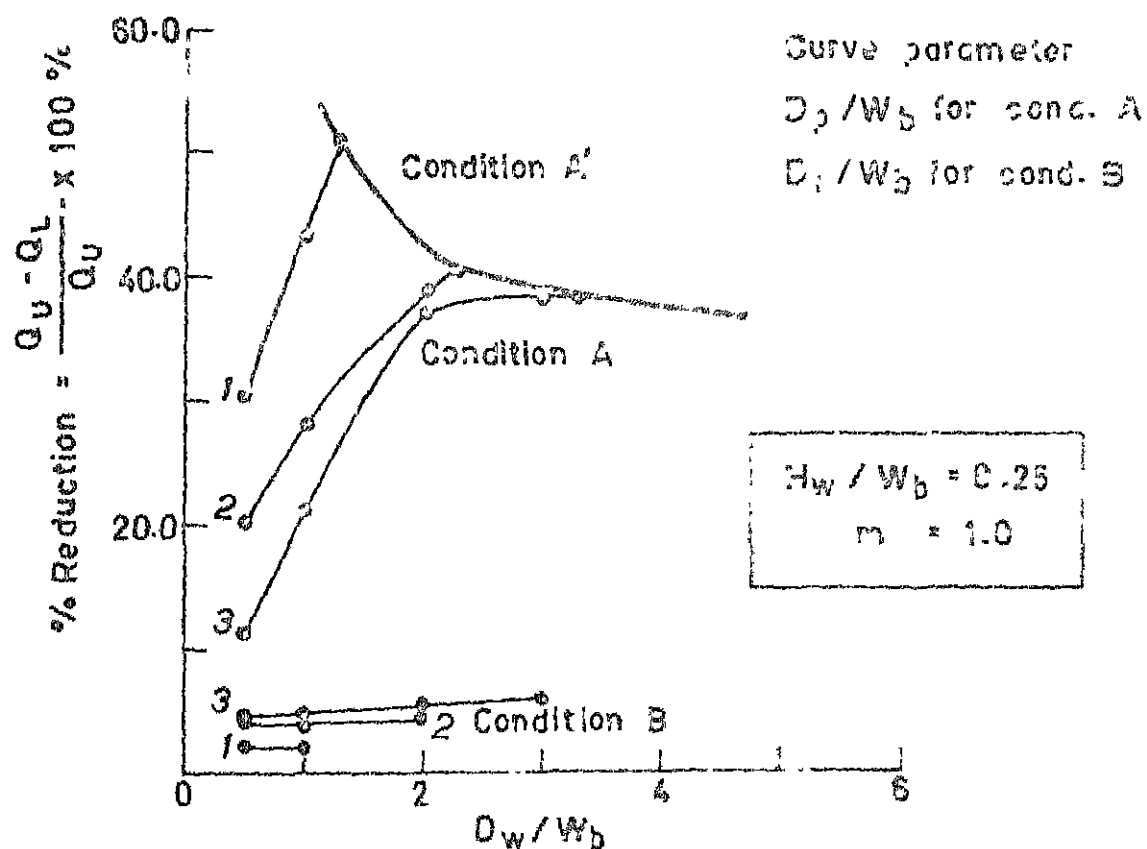
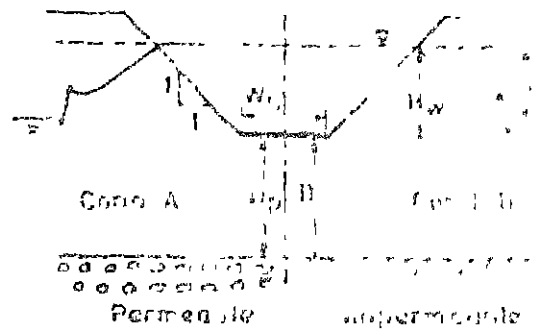
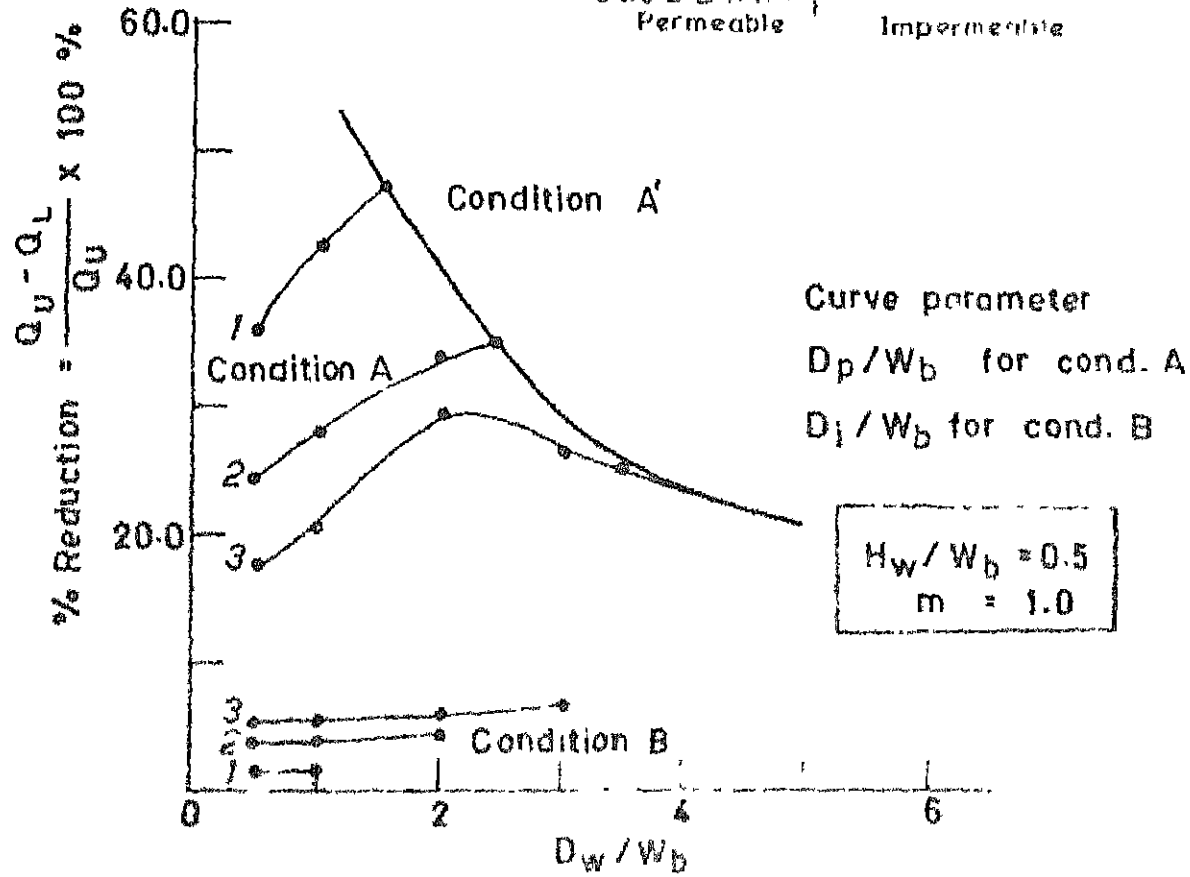
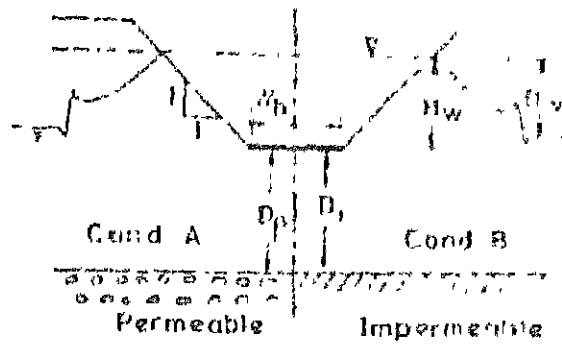


FIG. 5.9a. RESULTS OF SEEPAGE ANALYSIS FOR BOTTOM LINING OF CHANNEL WITH  $H_w / W_b = 0.25$  AND  $m = 1.0$





16. 5.9b. RESULTS OF SEEPAGE ANALYSIS FOR BOTTOM LIVING OF CHANNEL WITH  $H_w / W_b = 0.5$  AND  $m = 1.0$

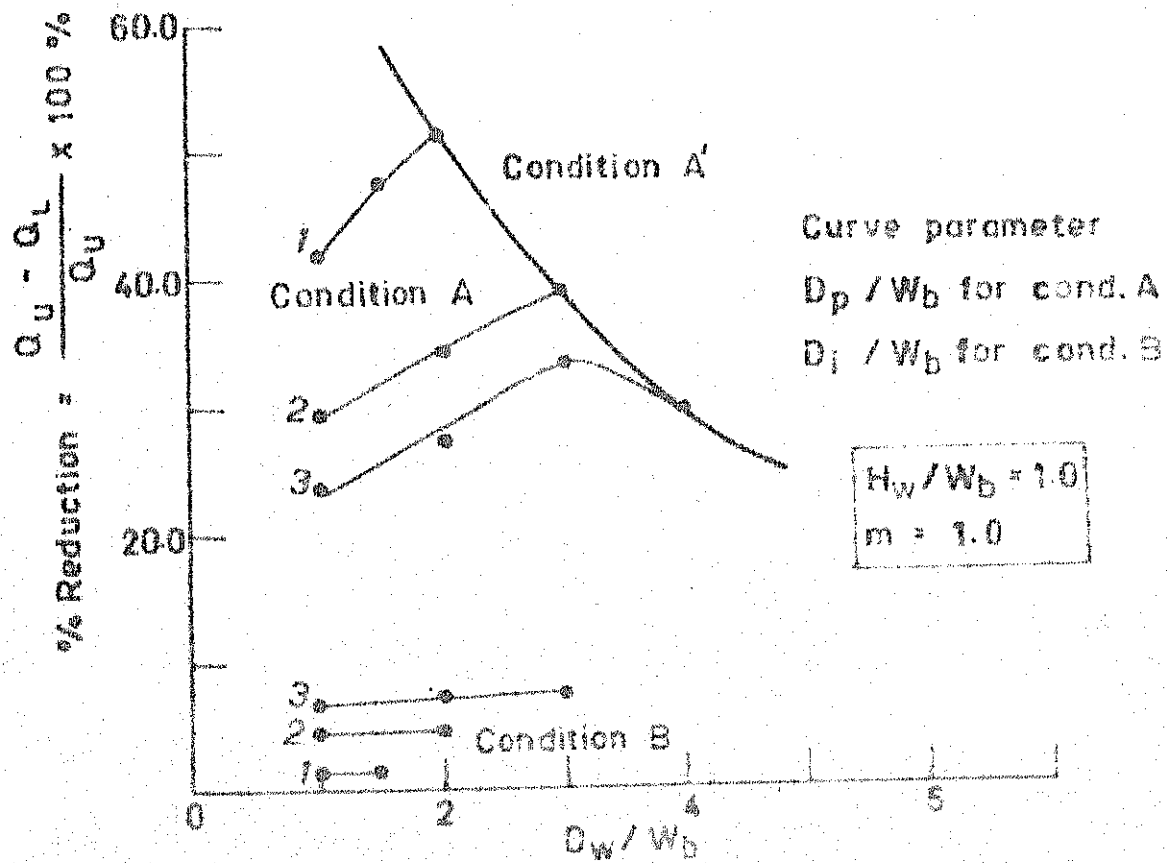
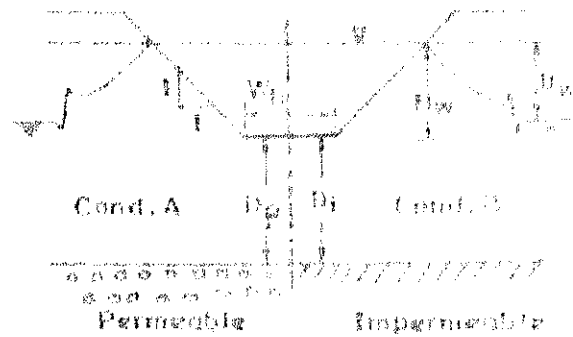


FIG 5.9c. RESULTS OF SEEPAGE ANALYSIS FOR BOTTOM LINING OF CHANNEL WITH  $H_w / W_b = 1.0$  AND  $m = 1.0$

TABLE 5.3

Results of Seepage analysis for base lined and sides not lined case  
 $W_b = 3.0$  mts. for condition A       $W_b = 6.0$  mts. for condition B  
 (or as indicated)

$$H_w/W_b = 0.25 \quad (W_b = 2.0 \text{ mts.})$$

Results for Condition A					Results for Condition B				
$D_p/W_b$ or $D_1/W_b$	$D_w/W_b$	Seepage Qty. for UL channel $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	Seepage Qty. for BL channel $Q_L (m^3/s)$	% Redu- ction	Lining Factor ( $f_L$ )	Seepage Qty. for UL channel $Q_U (m^3/s)$ ( $\times 10^{-2}$ )	Seepage Qty. for BL channel $Q_L (m^3/s)$ ( $\times 10^{-2}$ )	% Redu- ction	Lining Factor ( $f_L$ )
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1.0	0.5	2.016	1.411	30.01	-0.530	0.113	0.110	2.13	-1.138
	1.0	3.716	2.110	43.21	-0.241	0.173	0.169	2.19	-0.335
	0.5	1.326	1.061	20.00	-0.748	0.210	0.202	3.95	-1.098
2.0	1.0	2.456	1.782	27.50	-0.584	0.373	0.357	4.18	-1.094
	2.0	4.248	2.634	38.00	-0.355	0.557	0.532	4.50	-1.087
	0.5	1.101	0.980	10.99	-0.945	0.290	0.277	4.52	-1.086
3.0	1.0	2.069	1.634	21.02	-0.725	0.531	0.504	5.08	-1.074
	2.0	3.437	2.165	37.00	-0.376	0.873	0.825	5.50	-1.065
	3.0	4.738	2.914	38.50	-0.345	0.105	0.098	6.67	-1.043
$H_w/W_b = 0.50$									
1.0	0.5	2.308	1.470	36.31	-0.087	0.437	0.430	1.51	-0.681
	1.0	4.245	2.471	41.78	0.006	0.723	0.712	1.59	-0.679
	0.5	1.555	1.177	24.31	-0.292	0.740	0.709	4.16	-0.635
2.0	1.0	2.896	2.077	28.28	-0.224	1.337	1.280	4.27	-0.634
	2.0	5.011	3.311	33.93	-0.128	2.116	2.023	4.40	-0.632

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3.0	0.5	1.279	1.47	18.14	-0.397	0.982	0.928	5.47	-0.614
	1.0	2.416	1.940	19.70	-0.371	1.836	1.730	5.77	-0.608
	2.0	4.125	2.890	29.94	-0.196	3.127	2.935	6.14	-0.602
	3.0	5.639	4.142	26.54	-0.254	3.764	3.505	6.88	-0.589
$H_w/W_b = 1.0$									
1.0	1.0	5.481	3.181	41.96	0.214	1.088	1.073	1.37	-0.335
	1.5	6.793	3.566	47.50	0.289	1.657	1.632	1.53	-0.334
2.0	1.0	3.685	2.599	29.47	0.059	1.719	1.640	4.59	-0.292
	2.0	6.608	4.344	34.26	0.109	2.805	2.669	4.85	-0.288
3.0	1.0	2.987	2.279	23.70	-0.043	2.250	2.095	6.88	-0.261
	2.0	5.444	3.959	27.28	0.015	3.876	3.588	7.43	-0.253
	3.0	7.213	4.772	33.84	0.104	4.951	4.577	7.55	-0.252
$H_w/W_b = 2.0$									
1.0	2.0	13.900	7.109	48.86	0.398	3.137	3.086	1.63	-0.165
	2.5	15.746	7.601	51.73	0.512	4.789	4.697	1.92	-0.158
2.0	2.0	9.267	6.207	33.02	0.206	4.447	4.253	4.36	-0.126
	3.0	12.890	7.869	38.96	0.189	5.688	5.420	4.71	-0.121
3.0	2.0	7.492	5.460	27.12	0.189	5.651	5.243	7.22	-0.092
	3.0	10.047	7.154	31.67	0.196	7.463	6.904	7.49	-0.089

CONTD ....TABLE 5.3

For Condition A'						
$\frac{H_w}{W_b}$	$\frac{D_p}{W_b}$	$\frac{D_w}{W_b}$	Width of base $W_b$ (m)	Seepage Qty. for UL chan- nel $Q_U$ (m <sup>3</sup> /s) $\times 10^{-2}$	Seepage Qty. for BL channel $Q_L$ (m <sup>3</sup> /s) $\times 10^{-2}$	Percentage reduction  Lining Factor ( $f_L$ )
0.25	1.0	1.25	2.0	2.400	1.157	-0.164
	2.0	2.25	2.0	2.387	1.421	-0.437
	3.0	3.25	2.0	2.321	1.429	-0.486
	1.0	1.50	3.0	4.907	2.572	0.105
0.50	2.0	2.50	3.0	5.306	3.431	-0.104
	3.0	3.50	3.0	5.705	4.267	-0.277
	1.0	2.00	3.0	9.065	4.396	0.344
1.0	2.0	3.00	3.0	7.426	4.496	0.180
	3.0	4.00	3.0	7.250	5.071	0.053
	1.0	3.00	3.0	17.520	7.996	0.163
	2.0	4.00	3.0	15.060	8.355	0.347
2.0	3.0	5.00	3.0	14.760	8.810	0.298

the channel are lined as can be seen from Fig. 5.7 a,b and c.

#### 5.8.2 Seepage for Channels with Base Lined and Sides Unlined:

Seepages for the channel with base lined for the complete range of  $D_1$  and  $D_w$  for condition B and of  $D_p$  and  $D_w$  for condition A were obtained. The base lining of the channel was achieved by specifying zero nodal discharges at nodes 1,2 and 3 of the FE mesh (Fig. 5.5) . The conditions A and B were simulated as for the channels with sides lined, The position where the phreatic surface met the GWT was taken as ten times the width of the base of the channel. The results are presented in the form of non-dimensional parameters same as those used for the sides lined channels. Typical flow systems for conditions B, A and A' are shown in Fig. 5.8. The percentage reduction in seepage obtained for each of the set of parameters and condition are given in Table 5.3. The results are presented in the form of graphs in Fig. 5.9 a,b,c. Condition A' represents  $D_w = D_p + H_w$ . Further lowering of  $D_w$  does not increase the effective value of  $D_w$ . Thus, for condition A' the  $D_w$  values at the abscissa should be interpreted as  $D_p + H_w$ .

The percentage reduction in seepage due to lining of the base for condition B is very small as seen from Fig.5.9 and Table 5.3 as compared to the percentage reductions for condition A. The effect of the position of water table ( $D_w$ ) is marginal on percentage reduction in seepage for condition B.

Thus, the effect of water table position on the percentage reduction in seepage can be neglected for condition B. However, the effect of  $D_w$  on percentage reduction of seepage for condition A is significant and the percentage reduction tends to increase with increase in  $D_w/W_b$ .

The depth of impermeable layer ( $D_1$ ) for condition B affects the percentage reduction in seepage by increasing it when  $D_1/W_b$  itself is also increased. On the other hand, an increase in  $D_p/W_b$  for condition A tends to decrease the percentage reduction in seepage.

#### 5.9 Instances of the Development of Phreatic Surface at the Base of the Channel:

When the base of the channel is lined with an impermeable material, there is a possibility of the flow line separating from the base and developing into a phreatic surface. This phenomenon, however, is expected to depend upon the permeability of the medium in which the channel is laid ( $k$ ), the width of the base ( $W_b$ ), the position of the GWT ( $D_w$ ) and the position of impermeable ( $D_1$ ) or permeable layer ( $D_p$ ) underlying the channel. During the present study it was observed that for condition B, there was no instance when the phreatic surface was developed. This fact was further explored by increasing  $W_b$ ,  $D_1$  and  $D_w$  and decreasing  $k$  but still the base phreatic surface did not develop. On the other hand, for condition A, for the normal working dimensions and values,

of parameters , same as those used for condition B, the base phreatic surface promptly developed.

Therefore, conclusion can be drawn that though one might expect the development of base phreatic surface when the base of a channel is lined, its generation is significantly affected by the permeability of the underlying layer of soil.

#### 5.10 Alternate Way of Representation of Results:

Though partial lining of the channels leads to some amount of reduction in seepage, the reduction is not proportional to the percentage of the wetted perimeter of channel lined. That is, if, say, 60 percent of the channel perimeter is lined it is not necessary that reduction in seepage should also be 60 percent. Often it is less. This fact is highlighted by obtaining a factor called the lining factor ( $f_L$ ). The lining factor is the ratio of the change in seepage quantity per unit unlined wetted perimeter of channel to the seepage quantity per unit wetted perimeter of unlined channel. Thus,

$$f_L = \frac{Q_U/P_T - Q_L/(P_T - P_L)}{Q_U/P_T} \quad (5.4)$$

where,

$f_L$  = Lining factor

$Q_U$  = Seepage quantity for unlined channel



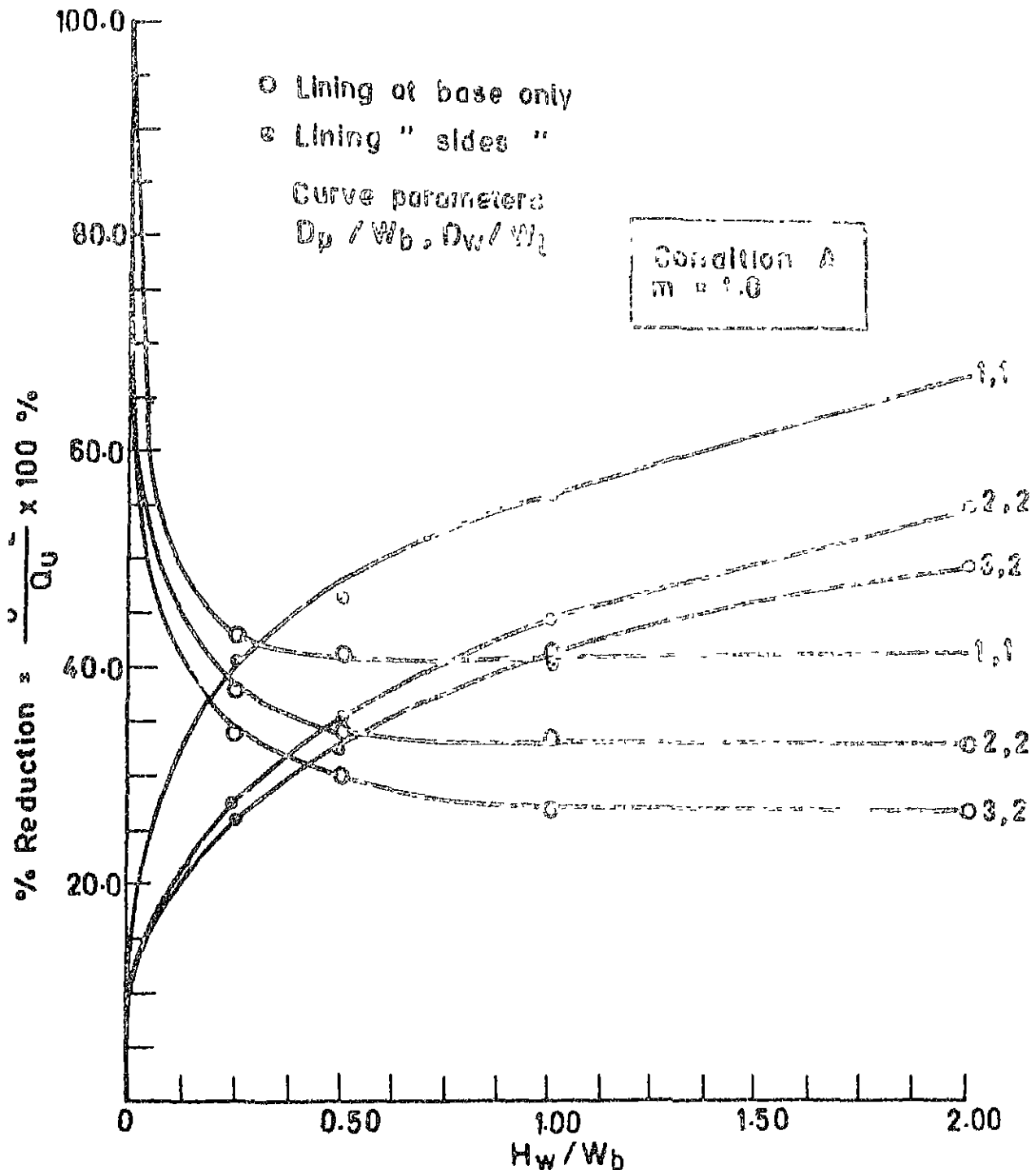


FIG. 5.10 PERCENTAGE REDUCTION IN SEEPAGE BY LINING THE SIDES AND BOTTOM SEPARATELY FOR AN UNDERLYING PERMEABLE LAYER

$Q_L$  = Seepage quantity for partially lined channel

$P_T$  = Total wetted perimeter of unlined channel

$P_L$  = Total wetted unlined portion of channel.

For a linear relationship between the percentage reduction in seepage and the percentage of the lined portion of the channel, the value of  $f_L$  should be zero. However, if the percentage reduction is less than the proportion of lining than the value of  $f_L$  would be negative. Thus,  $f_L$  greater than zero represents a cost efficient channel while  $f_L$  less than zero represents a cost inefficient channel. Table 5.2 and 5.3 represent the lining factors for all set of parameters and conditions and cases of lining obtained by this study.

#### 5.11 Comparative Performance of the Two Types of Partial Linings:

From the results of the present study presented in section 5.8 for the base lined and sides lined cases, it is apparent that there is substantial reduction in seepage (ranging from 30-55 percent) due to lining of either base or sides only for condition A as compared to the reductions in seepage for condition B (ranging from 2-15 percent) for the range of parameters considered. Owing to this fact, the curves represented in Fig. 5.10 are prepared for the partial lining cases for condition A only.

The curves of Fig. 5.10 represent the percentage reduction in seepage due to partial lining of the channel

versus the depth of water in the channel (represented non-dimensionally as  $H_w/W_b$ ) for various values of  $D_p/W_b$ . For each set of curve parameters, the curves of sides lined only and base lined only cases are obtained. These curves intersect at one point for each set of parameters. The point of intersection is very crucial for deciding the depth of water in the channel such that for  $H_w/W_b$  greater than the value for this depth, sides lining is more effective than base lining and for  $H_w/W_b$  less than this value base lining is more effective than sides lining in reducing seepage. From the curves of Fig. 5.10, a value of about 0.40 for  $H_w/W_b$  is obtained for  $D_p/W_b$  varying from 2 to 3 and  $\frac{D_w}{W_b} = 2$  such that for  $H_w/W_b$  less than 0.40 the lining of the base only would be more effective than lining the sides. The percentage reduction achievable is greater than 33% depending upon the  $H_w/W_b$  value. However for  $H_w/W_b$  greater than 0.40 sides lining would be more effective in reducing seepage and the percentage reduction is greater than 33% depending upon the value of  $H_w/W_b$ .

Since the curves shown in Fig. 5.10 demarcate the limits of efficiency of the base lined and sides lined cases of partial lining of a channel, their importance cannot be overemphasized while deciding the type of partial lining for a channel. For a given set of conditions of  $D_p$ ,  $D_w$ ,  $H_w$  and  $W_b$ , the curves help to obtain the most economical solution subject to the condition of a given percentage reduction in seepage.

## CHAPTER 6

## CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion:

With transit losses in irrigation channels upto 45 percent of the total intake at the canal head, the importance of lining of the channels cannot be overemphasized. Though considerable research has been done on analysis of seepage through unlined channels, there was no literature available on analysis of seepage through partially lined channels except certain observations reported by Bouwer [11,12] and Brebbia [28]. Thus a need was felt for further study in this area.

The analysis of seepage through partially lined channels for a set of different variables that govern seepage was done using a program 'CANSEP-86', initially developed by Achar [23] but was modified for present use, based on finite element analysis. Before being put to use for the present study the results obtained by the program were verified with works of Bouwer [11,12] on seepage through unlined channels.

Steady state two-dimensional unconfined seepage through channels depends upon a host of factors and though all such variables and various boundary conditions can be taken care of by the finite element method of analysis, the actual physical situation was approximated by two conditions

A and B keeping in view the general nature of this study. The side slope of 1H : 1V was kept constant in the present study. The following significant conclusions based on the results of analysis performed for base lined only and sides lined only cases for each of conditions A and B are obtained:

1. It is observed that the condition A is more active than condition B and therefore larger quantity of seepage as well as higher reduction in seepage is observed for condition A for the cases of both base lined only and sides lined only.

2. There is very marginal effect of the position of water table ( $D_w$ ) on reduction in seepage through base lined only as well as side only lined channels <sup>for</sup> condition B and therefore can as well be neglected while analysing seepage reduction.

3. The percentage reduction in seepage decreased with increase in the depth of infinitely permeable layer ( $D_p$ ) for condition A while increased with increase in the depth of impermeable layer ( $D_i$ ) for condition B, and, as expected, the results show a trend of approaching a common value of percentage reduction in seepage for both conditions A and B when  $D_i$  and  $D_p$  approach infinity.

4. The percentage reduction in seepage is not found to be directly proportional to the percentage of the wetted perimeter lined for partially lined channels. For

deeper channels ( $H_w/W_b=2$ ) lining factor as low as - 2 has been observed for sides only lined cases for condition A thereby severely affecting their cost efficiency though sides lining only may reduce seepage to the order of 60 percent.

5. The development of bottom phreatic surface was not observed for any of the cases for condition B when the base only was lined while it was readily apparent for condition A, thereby indicating the role played by the permeability of the underlying layer.

6. For condition A, for  $D_p/W_b=2$  and  $D_w/W_b=2$  and for  $D_p/W_b=3$  and  $D_w/W_b=2$  it is observed that for  $H_w/W_b$  less than 0.4 base lining only of the channel is more effective in reducing the seepage than side lining only. However, for  $H_w/W_b$  greater than 0.4 sides lining only is more effective in reducing seepage than base lining only. The order of magnitude of percentage reduction in seepage for  $H_w/W_b=0.4$  is 33. It may be as high as 70-80 percent for base lining only case when  $H_w/W_b$  is less than 0.06 and upto 55 percent for the case of sides only lined for condition A when  $H_w/W_b$  is greater than 2.

## 6.2 Recommendations:

1. As has been shown by the present study, the set of curves obtained for different cases of lining, namely, base only lined and sides only lined, representing the relationship between the percentage reduction in seepage and

$H_w/W_b$  are very helpful in deciding the limits of the efficiencies of each of the lining cases for a set of different values of parameters in terms of  $H_w/W_b$ . It would be worthwhile to obtain more such sets of curves for large ranges of values of the parameters governing seepage so as to be useful for practical purposes.

2. The side slopes of the channel were kept constant (1H:1V) for the present study. The analysis may also be performed for various values of side slopes which are generally provided like 1.5H:1V, 2H:1V etc.

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## APPENDIX I

## PROGRAM USER INSTRUCTIONS

The following assumptions were made for this program.

1. The positive direction of x is from left to right and y is positive upwards with the underlying layer, permeable or impermeable, as the datum.
2. The principal axes of seepage coincide with the x and y axes.
3. The units for length and time are in meters and seconds respectively. The pressure is in terms of the head in meters of water above the bottom layer, while the discharge is in cumecs.

All the input data are format free. Numbers must be separated by one or more blank spaces or by a comma. The program is written in FORTRAN -IV.

The general instructions are described below. Two different examples given at the end further clarify the procedure.

### I. General User Instructions:

Set of  
Input  
Data

Data

- |   |   |
|---|---|
| 1 | (i) INRG: Total number of superelements in which the area of analysis is divided. |
|   | (ii) INBP: Total number of superelement nodes.                                    |

1 (ctd.) (vii) IRENUM: Renumbering of the node numbers and the elements.

= 1 Renumber the mesh.

= 0 Do not renumber the mesh..

(viii) IBOTLN: Whether or not the channel is lined at the bottom?

= 1 Bottom of the channel is not lined.

= other Bottom is lined.

(ix) IDIFIM: Whether there are different impermeabilities in the area of analysis?

= 1 Soil is homogenous.

= other Soil is not homogenous.

(x) IADASH: Whether water table is at or below the permeable layer?

= 1 Water table is not at or below the permeable layer.

= other Water table is at or below the permeable layer.

2 ( To be read if IPOL is not equal to zero)

(i) IPOLAR: Polar co-ordinate reference number.

(ii) XOFSET (IPOLAR): Offset of the origin of the polar co-ordinate system in x-direction with reference to the global co-ordinate's origin.

(iii) YOFSET (IPOLAR): offset in the y-direction of the origin of the polar co-ordinate system with reference to the global co-ordinate's origin.

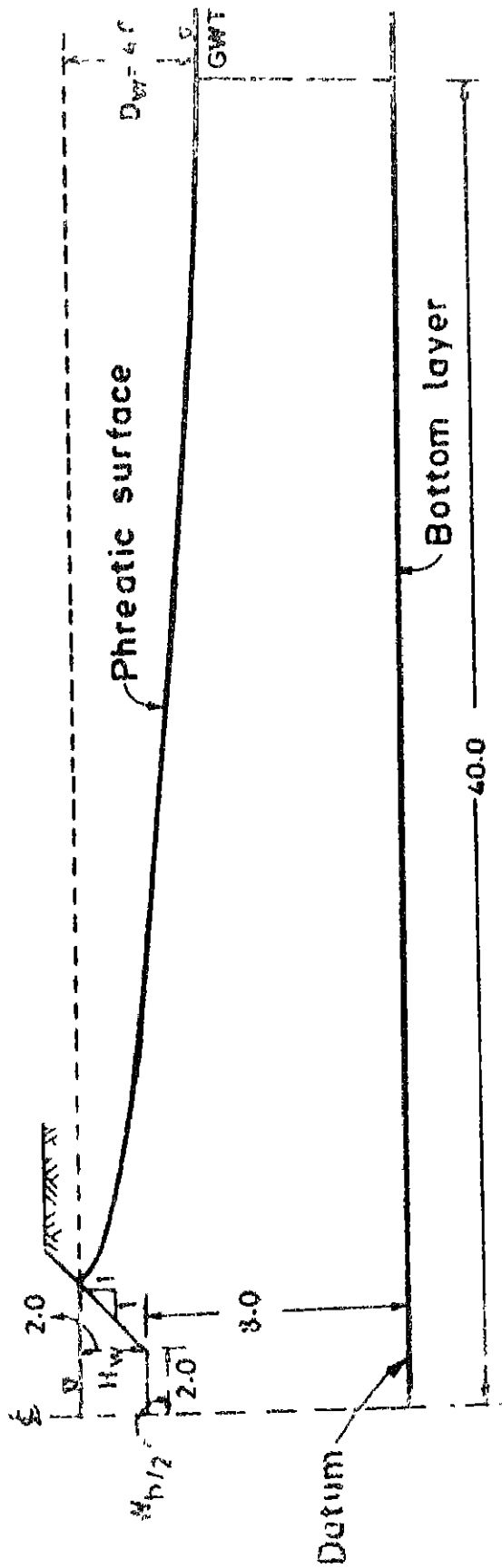
- 3        (1)    IPOLAR: Co-ordinate reference.  
              = 0 Cartesian co-ordinate system.  
              = 1 Polar co-ordinate system reference  
                      number 1.  
              .  
              .  
              .  
              = N Polar co-ordinate system reference  
                      number N.
- (ii)   K: Superelement node number. (K=1,INBP)
- (iii)   XP(K,1): X co-ordinate of the node if  
                      IPOLAR = 0 else radius of the node position  
                      from the origin at the datum.
- (iv)   YP(K,1): Y co-ordinate of the node if IPOLAR=0  
                      else angle of the node position from the  
                      origin at the datum.
- 4 ( To be read only if ITMP is not equal to zero)
- (i)    K: Superelement node number (K=1,INBP)
- (ii)   XP(K,2): The nodal temperature at the super-  
                      element nodes.
- 5 ( To be read only if ITHK is not equal to zero)
- (i)    K: Superelement node number. (K=1,INBP)
- (ii)   YP (K,2): The nodal thickness.
- 6 ( To generate the elements)
- (i)    NRG: Number of the superelement.
- (ii)   NRW1(NRG): Number of divisions in the  
                      superelement columnwise.

- 6 (ctd.) (iii) NCOL1(NRG): Number of divisions in the  
superelement rowwise.
- (iv) IND1: Indicates the total number of boundary  
nodes for the given superelement.  
= 1 Total number of boundary nodes  
equal to eight.  
= 2 Total number of boundary nodes  
equal to four.
- (v) NOP(NRG,K),K=1,8,IND1: The superelement  
node numbers read consecutively and in  
counter-clockwise direction.
- (vi) NMAT(NRG): Material code number for this region.
- 7 (i) TITLE: Any title for specifying the problem  
(maximum 16 characters).
- 8 (i) UNIT: Units in which various quantities  
are represented.  
= 1 M.K.S. System.  
= 2 F.P.S. System.
- 9 (i) TYPE: Type of the media.  
= 1 Homogenous and isotropic porous media.  
= 2 Non-homogenous and anisotropic porous  
media.  
= 3 When the phreatic surface traverses  
porous media of different hydraulic  
conductivities.



- 10        (i)    NNI: Total number of nodes of the finite element mesh.
- (ii)   NE : Total number of elements into which the region is divided.
- (iii)   NST: Total number of nodes at which the head is specified.
- (iv)   NSQ: Total number of nodes at which flux is specified.
- 11        (i)    QQQ: Flow generated in the element initially.
- 12        (i)    NT: Number of the node which has head specified.
- (ii)   TNT: The head which is specified at a certain node (NT).
- (NT and TNT are read in pairs till the total number of pairs is equal to NST).
- 13    ( To be read only if IMPDIF is not equal to 1)
- (i)    TKU: Permeability in the upper (right) region of the area of analysis.
- (ii)   TKD: Permeability in the lower (left) region of the area of analysis.
- 14        (i)    LIMITL: The node number such that all node numbers lesser than it have permeability TKU and greater than it have permeability TKD.
- (ii)   LIMITU: The node number such that all node numbers greater than it have permeability TKU and lesser than it have permeability TKD.

- 15 (To be read if IMPDIF is equal to 1)
- (i) TKX (I), I= 1,NE: Permeability in x-direction.
  - (ii) TKY (I), I= 1,NE: Permeability in y-direction
- 16 (i) NFS: Total number of phreatic surface nodes.
- 17 (i) NFY (I), I= 1,NFS: Node numbers,related to  
the initial super elements,of the nodes  
lying on the phreatic surface.
- (ii) NFT (I), I= 1,NFS: Node numbers, corresponding  
to the finite element mesh, of the nodes  
lying on the phreatic surface.
- 18 (To be read if IBOTLN is not equal to 1)
- (i) NOBNOD: Total number of nodes lying on the  
bottom of the channel.
  - (ii) DWATBL: Depth of the GWT from the datum.
- 19 (i) NBPHY (I),I= 1,NOBNOD: Node numbers,  
related to initial superelements, of the  
nodes lying on the bottom of the channel.
- (ii) NBPHT (I), I= 1,NOBNOD: Node numbers,  
corresponding to the finite element mesh,  
of the nodes lying on the bottom of the  
channel.



Example 1 > Bottom layer : Impermeable

Lining : Sides only ; Base unlined

Example 2 > Bottom layer : Permeable

Lining : Base only ; Sides unlined

*All dimensions in meters*

FIG. A-1. SCHEMATIC DIAGRAM OF EXAMPLE 1 AND 2



18	2.20846	2.20846
19	1.09592	1.09592
20	1.09418	1.09418
21	0.00000	0.00000
22	0.00000	0.00000
23	7.77795	7.77795
24	7.32866	7.32866
25	6.10824	6.10824
26	5.84925	5.84925
27	4.73053	4.73053
28	4.47304	4.47304
29	3.37686	3.37686
30	3.20300	3.20300
31	2.13721	2.13721
32	2.03015	2.03015
33	1.01159	1.01159
34	0.96348	0.96348
35	0.00000	0.00000
36	0.00000	0.00000
37	6.74673	6.74673
38	6.00000	6.00000
39	5.49335	5.49335
40	5.00000	5.00000
41	4.29149	4.29149
42	4.00000	4.00000
43	3.14123	3.14123
44	3.00000	3.00000
45	2.04256	2.04256
46	2.00000	2.00000
47	0.99548	0.99548
48	1.00000	1.00000
49	0.00000	0.00000
50	0.00000	0.00000

Node numbering Generated by AUTO-MESH GENERATION SCHEME  
 =====

Element	Material	Nodes
1	1	4 5 2
2	1	4 5 3
3	1	5 7 3
4	1	7 7 8
5	1	8 8 6
6	1	10 11 8
7	1	10 11 12
8	1	11 13 14
9	1	13 13 11
10	1	13 15 12
11	1	14 12 17
12	1	14 17 14
13	1	16 14 15
14	1	16 15 17
15	1	17 17 15
16	1	19 20 17
17	1	19 20 18
18	1	20 24 18
19	1	24 22 25
20	1	24 25 23
21	1	25 23 22
22	1	22 23 22











61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130																																																												
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	1																						

131	.671E+04	-.100E+00	-.450E-03	.000E+00	-.741E-04	.000E+00
132	.671E+04	-.153E-03	-.000E-03	.000E+00	-.453E-03	-.153E-03
133	.671E+04	-.716E-11	-.537E-03	-.900E-04	-.600E-05	.000E-04
134	.671E+04	-.486E-04	-.565E-03	-.344E-11	-.537E-03	-.356E-04
135	.671E+04	-.152E-13	-.500E-03	-.238E-08	-.222E-04	-.238E-08
136	.671E+04	-.155E-03	-.576E-03	-.716E-11	-.502E-03	-.405E-03
137	.671E+04	-.155E+00	-.523E-03	-.520E-04	-.155E-01	-.525E-04
138	.671E+04	-.509E-04	-.515E-03	.000E+00	-.521E-03	-.509E-04
139	.671E+04	-.152E-15	-.523E-03	-.238E-08	-.235E-05	-.238E-08
140	.671E+04	-.611E-04	-.547E-03	.000E+00	-.524E-03	-.614E-04
141	.671E+04	-.138E-11	-.481E-03	-.173E-04	.000E+00	-.173E-04
142	.671E+04	-.168E-04	-.465E-03	-.669E-12	-.481E-03	-.168E-04
143	.671E+04	.000E+00	-.520E-03	.000E+00	.000E+00	.000E+00
144	.671E+04	-.202E-04	-.516E-03	-.138E-11	-.521E-03	-.202E-04

## Coordinates of the Phreatic Surface Nodes

Y coordinates of PS nodes	Heads of PS nodes
9.238	9.209
9.235	9.041
6.744	8.791
1.499	8.506
5.123	8.132
7.786	7.789
7.349	7.337
6.747	6.756
6.293	6.400

Total Seepage = 2.\* 0.0073 Cume/s

' J u n e ' ,

Refer to the definitions of the  
variables given below:

$\Delta x = 1.0 \text{ mts.}$  ,  $\Delta y = 1.0 \text{ mts.}$  ,  $\Delta z = 1.0 \text{ mts.}$

12345678910111213141516171819202122232425262728293031323334353637383940414243444546474849505152535455565758596061626364656667686970717273747576777879808182838485868788899091929394959697989910010110210310410510610710810911011111211311411511611711811912012112212312412512612712812913013113213313413513613713813914014114214314414514614714814915015115215315415515615715815916016116216316416516616716816917017117217317417517617717817918018118218318418518618718818919019119219319419519619719819920020120220320420520620720820921021121221321421521621721821922022122222322422522622722822923023123223323423523623723823924024124224324424524624724824925025125225325425525625725825926026126226326426526626726826927027127227327427527627727827928028128228328428528628728828929029129229329429529629729829930030130230330430530630730830931031131231331431531631731831932032132232332432532632732832933033133233333433533633733833934034134234334434534634734834935035135235335435535635735835936036136236336436536636736836937037137237337437537637737837938038138238338438538638738838939039139239339439539639739839940040140240340440540640740840941041141241341441541641741841942042142242342442542642742842943043143243343443543643743843944044144244344444544644744844945045145245345445545645745845946046146246346446546646746846947047147247347447547647747847948048148248348448548648748848949049149249349449549649749849950050150250350450550650750850951051151251351451551651751851952052152252352452552652752852953053153253353453553653753853954054154254354454554654754854955055155255355455555655755855956056156256356456556656756856957057157257357457557657757857958058158258358458558658758858959059159259359459559659759859960060160260360460560660760860961061161261361461561661761861962062162262362462562662762862963063163263363463563663763863964064164264364464564664764864965065165265365465565665765865966066166266366466566666766866967067167267367467567667767867968068168268368468568668768868969069169269369469569669769869970070170270370470570670770870971071171271371471571671771871972072172272372472572672772872973073173273373473573673773873974074174274374474574674774874975075175275375475575675775875976076176276376476576676776876977077177277377477577677777877978078178278378478578678778878979079179279379479579679779879980080180280380480580680780880981081181281381481581681781881982082182282382482582682782882983083183283383483583683783883984084184284384484584684784884985085185285385485585685785885986086186286386486586686786886987087187287387487587687787887988088188288388488588688788888989089189289389489589689789889990090190290390490590690790890991091191291391491591691791891992092192292392492592692792892993093193293393493593693793893994094194294394494594694794894995095195295395495595695795895996096196296396496596696796896997097197297397497597697797897998098198298398498598698798898999099199299399499599699799899910001001100210031004100510061007100810091010101110121013101410151016101710181019102010211022102310241025102610271028102910301031103210331034103510361037103810391040104110421043104410451046104710481049105010511052105310541055105610571058105910601061106210631064106510661067106810691070107110721073107410751076107710781079108010811082108310841085108610871088108910901091109210931094109510961097109810991100110111021103110411051106110711081109111011111112111311141115111611171118111911201121112211231124112511261127112811291130113111321133113411351136113711381139114011411142114311441145114611471148114911501151115211531154115511561157115811591160116111621163116411651166116711681169117011711172117311741175117611771178117911801181118211831184118511861187118811891190119111921193119411951196119711981199120012011202120312041205120612071208120912101211121212131214121512161217121812191220122112221223122412251226122712281229123012311232123312341235123612371238123912401241124212431244124512461247124812491250125112521253125412551256125712581259126012611262126312641265126612671268126912701271127212731274127512761277127812791280128112821283128412851286128712881289129012911292129312941295129612971298129913001

For this example, the seepage calculated is for the case that 100% storage for condition 2.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 1040 1

[illegible]

62 0.  
 63 0.  
 75 0.  
 77 0.  
 79 0.  
 81 0.  
 83 0.  
 85 0.  
 87 0.  
 89 0.  
 90 0.  
 91 0.  
 288 0.

11 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31  
 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100  
 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100  
 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100



18.22000	1.36859
18.22000	0.00000
18.22000	0.00000
22.22000	6.04262
27.22000	6.01831
22.22000	5.27427
27.22000	5.12129
22.22000	4.41042
27.22000	4.19185
22.22000	3.45156
27.22000	3.20000
22.22000	2.32621
27.22000	2.17571
22.22000	1.21585
27.22000	1.10909
22.22000	0.00000
27.22000	0.00000
33.22000	6.01177
33.22000	6.00000
33.22000	5.06282
40.22000	5.00000
33.22000	4.09265
33.22000	4.00000
33.22000	3.10131
40.22000	3.00000
33.22000	2.08871
40.22000	2.00000
33.22000	1.05497
40.22000	1.00000
33.22000	0.00000
40.22000	0.00000

Mapa Elementarj Generated by AUTO-MESH GENERATION SCHEME

Element	Material	Nodes
1	1	5
2	1	11
3	1	2
4	1	3
5	1	4
6	1	5
7	1	6
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63	1.14E-02	1.532E-03	1.00F+00	1.576F+04	1.304E-02
64	1.16E-02	1.693E-03	1.161E-02	1.723F+03	1.368E-10
65	1.17E-02	1.217E-03	1.295E-02	1.415E+01	1.388E-10
66	1.18E-02	1.535E-03	1.798E-10	1.415E+01	1.401E-10
67	1.19E-02	1.243E-03	1.585E-10	1.258E+04	1.295E-10
68	1.20E-02	1.898E-03	1.171E-02	1.552E+03	1.171E-10
69	1.21E-02	1.900E+00	1.288E-02	1.000E+00	1.681E-10
70	1.22E-02	1.259E-03	1.000E+00	1.248E+03	1.379E-10
71	1.23E-02	1.600E+00	1.177E-02	1.000E+00	1.377E-10
72	1.24E-02	1.314E-03	1.571E-10	1.333E+03	1.284E-10
73	1.25E-02	1.887E-04	1.246E-03	1.225E+03	1.284E-10
74	1.26E-02	1.379E-03	1.600E+00	1.904E+04	1.392E-10
75	1.27E-02	1.707E-04	1.795E-04	1.467E+04	1.735E-04
76	1.28E-02	1.110E-03	1.836E-11	1.817E+04	1.165E-03
77	1.29E-02	1.171E-03	1.445E-03	1.191E+03	1.443E-03
78	1.30E-02	1.390E-03	1.000E+00	1.181E+03	1.134E-03
79	1.31E-02	1.459E-04	1.177E-03	1.468E+04	1.177E-03
80	1.32E-02	1.147E-03	1.513E-10	1.101E+03	1.572E-03
81	1.33E-02	1.227E-03	1.642E-03	1.136E+03	1.642E-03
82	1.34E-02	1.431E-03	1.000E+00	1.256E+03	1.165E-03
83	1.35E-02	1.100E-03	1.277E+03	1.377E+04	1.277E-03
84	1.36E-02	1.164E-03	1.000E+00	1.119E+03	1.825E-03
85	1.37E-02	1.234E-03	1.105E-03	1.733E+04	1.816E-03
86	1.38E-02	1.420E-03	1.638E-10	1.297E-03	1.187E-03
87	1.39E-02	1.101E-03	1.370E-03	1.226E-04	1.370E-03
88	1.40E-02	1.163E-03	1.556E-10	1.128E-03	1.370E-03
89	1.41E-02	1.165E-03	1.943E-03	1.218E-03	1.195E-03
90	1.42E-02	1.349E-03	1.681E-10	1.228E+03	1.420E-03
91	1.43E-02	1.594E-04	1.445E-03	1.724E-03	1.445E-03
92	1.44E-02	1.139E-03	1.323E-10	1.118E+03	1.445E-03
93	1.45E-02	1.000E+00	1.102E-02	1.000E+00	1.102E-02
94	1.46E-02	1.212E-03	1.000E+00	1.192E+03	1.212E-03
95	1.47E-02	1.000E+00	1.489E-03	1.000E+00	1.489E-03
96	1.48E-02	1.854E-04	1.489E-03	1.788E-04	1.489E-03
97	1.49E-02	1.424E-04	1.225E-04	1.143E-04	1.225E-04
98	1.50E-02	1.478E-04	1.421E-11	1.333E-04	1.478E-04
99	1.51E-02	1.173E-04	1.803E-05	1.336E-04	1.173E-04
100	1.52E-02	1.183E-04	1.114E-11	1.356E-04	1.183E-04
101	1.53E-02	1.440E-04	1.585E-04	1.169E-04	1.440E-04
102	1.54E-02	1.561E-04	1.934E-11	1.433E-04	1.561E-04
103	1.55E-02	1.152E-04	1.210E-11	1.433E-04	1.152E-04
104	1.56E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
105	1.57E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
106	1.58E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
107	1.59E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
108	1.60E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
109	1.61E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
110	1.62E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
111	1.63E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
112	1.64E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
113	1.65E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
114	1.66E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
115	1.67E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
116	1.68E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
117	1.69E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
118	1.70E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
119	1.71E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
120	1.72E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
121	1.73E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
122	1.74E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
123	1.75E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
124	1.76E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
125	1.77E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
126	1.78E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
127	1.79E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
128	1.80E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
129	1.81E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
130	1.82E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
131	1.83E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04
132	1.84E-02	1.197E-04	1.344E-11	1.169E-04	1.197E-04

135	.000E+00	-.161E-11	-.303E-09	-.129E-04	-.285E-05	.129E-04
136	.000E+00	-.567E-04	-.151E-05	.225E-11	.406E-05	.567E-04
137	.000E+00	.000E+00	.714E-06	.000E+00	.699E-07	.000E+00
138	.000E+00	-.151E-04	-.108E-05	.103E-11	.969E-05	.151E-04
139	.000E+00	.000E+00	-.176E-05	-.151E-04	-.941E-07	.151E-04
140	.000E+00	-.559E-04	-.336E-05	.000E+00	.108E-05	.559E-04
141	.000E+00	.000E+00	-.189E-05	.000E+00	-.226E-07	.000E+00
142	.000E+00	-.177E-04	-.777E-06	.000E+00	.710E-05	.177E-04
143	.000E+00	.000E+00	.000E+00	-.164E-04	.000E+00	.164E-04
144	.000E+00	-.727E-04	-.183E-05	-.289E-11	.174E-05	.727E-04
145	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00	.000E+00
146	.000E+00	-.104E-04	-.411E-06	-.130E-11	.395E-05	.104E-04

### Coordinates of the Phreatic Surface Nodes

Y coordinates of PS nodes	Heads of PS nodes
------------------------------	----------------------

10.000	10.000
7.559	7.559
5.579	5.579
5.276	5.276
5.093	5.093
5.033	5.033
5.010	5.010
5.002	5.002
5.000	5.000

### Bottom Phreatic Line

.786E+01	.725E+01
.951E+01	.925E+01
.149E+01	.938E+01

Total Seepage = 2.4 0.0279 cumecs

CE-1986-M-HVS-SLE